

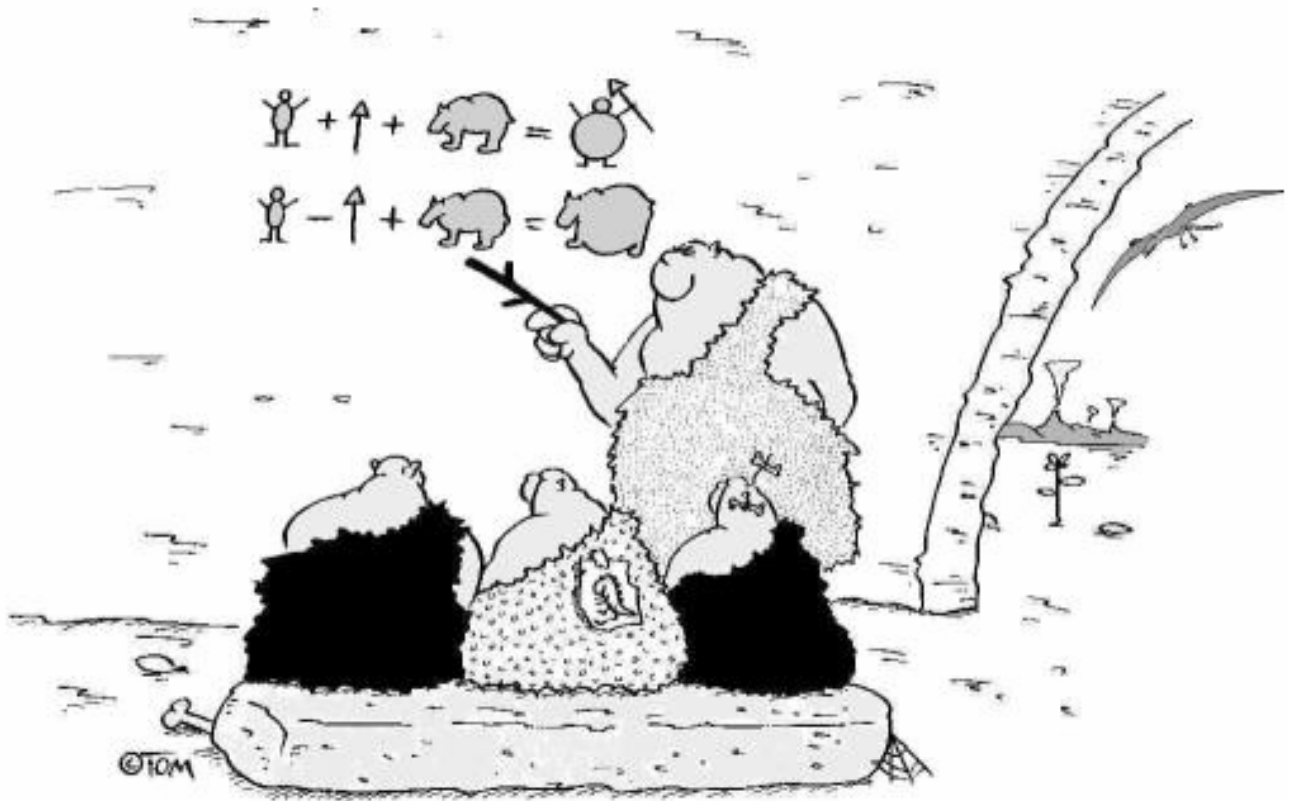
**Exploration, Direct Teach, and Assessment:
They can work together.**

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Early Colorado Mathematics



EXPLORATION

5-1

Identifying Linear Functions

The table shows the fees that a DVD rental company charges for movie rentals.

Movies	2	4	6	8	10
Cost (\$)	8	12	16	20	24

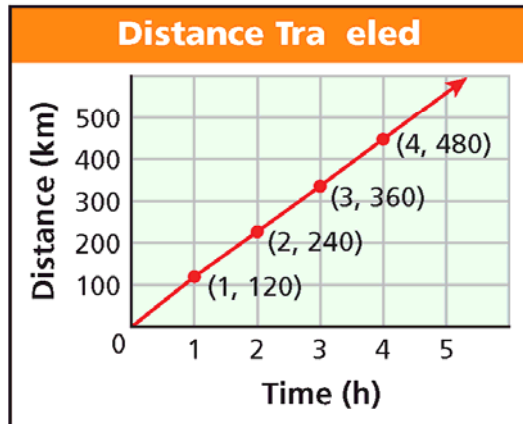
1. Which is the independent variable and which is the dependent variable?
2. For every increase of two movies, by how much does the cost increase?
3. Make a graph of the data.
4. What do you notice about the points that you plotted?

THINK AND DISCUSS

5. **Explain** how you know this relationship is a function.
6. **Describe** how to use your graph to predict the cost of renting 12 movies.

TEACHING TRANSPARENCY

5-1 Identifying Linear Functions

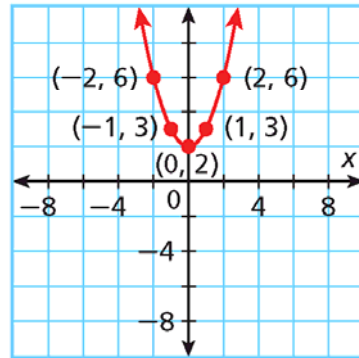
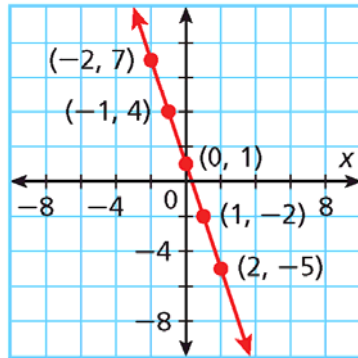



x	y
-2	7
-1	4
0	1
1	-2
2	-5

Vertical changes: +1, +1, +1, +1
Horizontal changes: -3, -3, -3, -3

x	y
-2	6
-1	3
0	2
1	3
2	6

Vertical changes: +1, +1, +1, +1
Horizontal changes: -3, -1, +1, +3



-  **52. Write About It** For $y = 2x - 1$, make a table of ordered pairs and a graph. Describe the relationships between the equation, the table, and the graph.
- 53. Critical Thinking** Describe a real-world situation that can be represented by a linear function whose domain and range must be limited. Give your function and its domain and range.




- 54.** This problem will prepare you for the Multi-Step Test Prep on page 332.
- Juan is running on a treadmill. The table shows the number of Calories Juan burns as a function of time. Explain how you can tell that this relationship is linear by using the table.
 - Create a graph of the data.
 - How can you tell from the graph that the relationship is linear?

Time (min)	Calories
3	27
6	54
9	81
12	108
15	135
18	162
21	189



- 38.** This problem will prepare you for the Multi-Step Test Prep on page 332.
- Kristyn rode a stationary bike at the gym. She programmed the timer for 20 minutes. The display counted backward to show how much time remained in her workout. It also showed her mileage.
- What are the intercepts?
 - What do the intercepts represent?

Time Remaining (min)	Distance Covered (mi)
20	0
16	0.35
12	0.70
8	1.05
4	1.40
0	1.75

-  **39. Write About It** Write a real-world problem that could be modeled by a linear function whose x -intercept is 5 and whose y -intercept is 60.



- 40.** Which is the x -intercept of $-2x = 9y - 187$

Area in the Coordinate Plane

Lines in the coordinate plane can form the sides of polygons. You can use points on these lines to help you find the areas of these polygons.

Example

Find the area of the triangle formed by the x -axis, the y -axis, and the line described by $3x + 2y = 18$.

Step 1 Find the intercepts of $3x + 2y = 18$.

x-intercept:	y-intercept:
$3x + 2y = 18$	$3x + 2y = 18$
$3x + 2(0) = 18$	$3(0) + 2y = 18$
$3x = 18$	$2y = 18$
$x = 6$	$y = 9$

Step 2 Use the intercepts to graph the line. The x -intercept is 6, so plot $(6, 0)$. The y -intercept is 9, so plot $(0, 9)$. Connect with a straight line. Then shade the triangle formed by the line and the axes, as described.

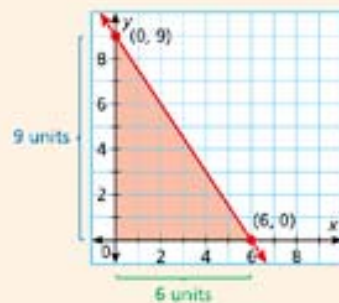
Step 3 Recall that the area of a triangle is given by $A = \frac{1}{2}bh$.

- The length of the base is 6.
- The height is 9.

Step 4 Substitute these values into the formula.

$$\begin{aligned}
 A &= \frac{1}{2}bh \\
 A &= \frac{1}{2}(6)(9) && \text{Substitute into the area formula.} \\
 &= \frac{1}{2}(54) && \text{Simplify.} \\
 &= 27
 \end{aligned}$$

The area of the triangle is 27 square units.



Try This

1. Find the area of the triangle formed by the x -axis, the y -axis, and the line described by $3x + 2y = 12$.
2. Find the area of the triangle formed by the x -axis, the y -axis, and the line described by $y = 6 - x$.
3. Find the area of the polygon formed by the x -axis, the y -axis, the line described by $y = 6$, and the line described by $x = 4$.

EXPLORATION

5-3 Rate of Change and Slope

An electronics company has several factories that make stereos. Each factory produces the stereos at a steady pace throughout the day.

1. Over a period of 8 hours, factory A produces 400 stereos. Complete the table.

Time (h)	1	2	3	4	5	6	7	8
Stereos Produced								400

2. Over a period of 6 hours, factory B produces 360 stereos. Complete the table.

Time (h)	1	2	3	4	5	6	7	8
Stereos Produced						360		

3. How many stereos does each factory produce per hour?
4. Graph a linear function for each of the factories on the same coordinate plane.
5. How do the graphs compare?

THINK AND DISCUSS

6. **Explain** how you found the number of stereos that each factory produces per hour.
7. **Discuss** how a graph for factory C, which produces 75 stereos per hour, would compare to the other factories' graphs.



Explore Constant Changes

There are many real-life situations in which the amount of change is constant. In these activities, you will explore what happens when

- a quantity increases by a constant amount.
- a quantity decreases by a constant amount.

Use with Lesson 5-3

Activity 1

Janice has read 7 books for her summer reading club. She plans to read 2 books each week for the rest of the summer. The table shows the total number of books that Janice will have read after different numbers of weeks have passed.

- 1 What number is added to the number of books in each row to get the number of books in the next row?
- 2 What does your answer to Problem 1 represent in Janice's situation? Describe the meaning of the constant change.
- 3 Graph the ordered pairs from the table. Describe how the points are related.
- 4 Look again at your answer to Problem 1. Explain how this number affects your graph.

Week	Total Books Read
0	7
1	9
2	11
3	13
4	15
5	17

Try This

At a particular college, a full-time student must take at least 12 credit hours per semester and may take up to 18 credit hours per semester. Tuition costs \$200 per credit hour.

1. Copy and complete the table by using the information above.
2. What number is added to the cost in each row to get the cost in the next row?
3. What does your answer to Problem 2 above represent in the situation? Describe the meaning of the constant change.
4. Graph the ordered pairs from the table. Describe how the points are related.
5. Look again at your answer to Problem 2. Explain how this number affects the shape of your graph.
6. Compare your graphs from Activity 1 and Problem 4. How are they alike? How are they different?
7. **Make a Conjecture** Describe the graph of any situation that involves repeated addition of a positive number. Why do you think your description is correct?

Credit Hours	Cost (\$)
12	■
13	■
14	■
15	■
16	■
17	■
18	■



Additional Examples

Example 1

The table shows the average temperature ($^{\circ}\text{F}$) for five months in a certain city. Find the rate of change for each time period. During which time period did the temperature increase at the fastest rate?

Month	2	3	5	7	8
Temp. ($^{\circ}\text{F}$)	56	56	63	71	72

The dependent variable is and the independent variable is .

month 2 to 3 $\frac{\text{change in temperature}}{\text{change in month}} = \frac{\square - \square}{\square - \square} = \frac{\square}{\square} = \frac{\square}{\text{month}}^{\circ}\text{F}$

month 3 to 5 $\frac{\text{change in temperature}}{\text{change in month}} = \frac{\square - \square}{\square - \square} = \frac{\square}{\square} = \frac{\square}{\text{month}}^{\circ}\text{F}$

month 5 to 7 $\frac{\text{change in temperature}}{\text{change in month}} = \frac{\square - \square}{\square - \square} = \frac{\square}{\square} = \frac{\square}{\text{month}}^{\circ}\text{F}$

month 7 to 8 $\frac{\text{change in temperature}}{\text{change in month}} = \frac{\square - \square}{\square - \square} = \frac{\square}{\square} = \frac{\square}{\text{month}}^{\circ}\text{F}$

The temperature increased at the greatest rate .

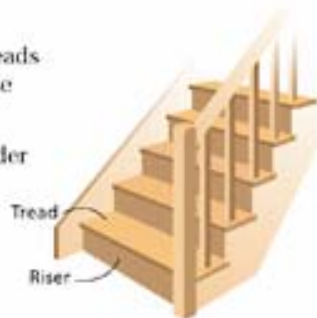
**MULTI-STEP
TEST PREP**



22. This problem will prepare you for the Multi-Step Test Prep on page 332.
- The graph shows a relationship between a person's age and his or her estimated maximum heart rate in beats per minute. Find the slope.
 - Describe the rate of change in this situation.



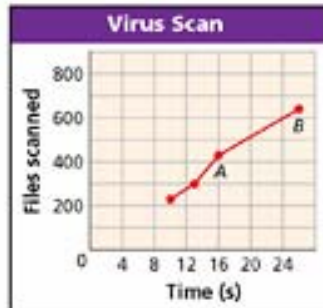
23. **Construction** Most staircases in use today have 9-inch treads and $8\frac{1}{2}$ -inch risers. What is the slope of a staircase with these measurements?
24. A ladder is leaned against a building. The bottom of the ladder is 9 feet from the building. The top of the ladder is 16 feet above the ground.
- Draw a diagram to represent this situation.
 - What is the slope of the ladder?



25. **Write About It** Why will the slope of any horizontal line be 0? Why will the slope of any vertical line be undefined?
26. The table shows the distance traveled by a car during a five-hour road trip.

Time (h)	0	1	2	3	4	5
Distance (mi)	0	40	80	80	110	160

- Graph the data and show the rates of change.
 - The rate of change represents the average speed. During which hour was the car's average speed the greatest?
27. **Estimation** The graph shows the number of files scanned by a computer virus detection program over time.
- Estimate the coordinates of point A.
 - Estimate the coordinates of point B.
 - Use your answers from parts a and b to estimate the rate of change (in files per second) between points A and B.



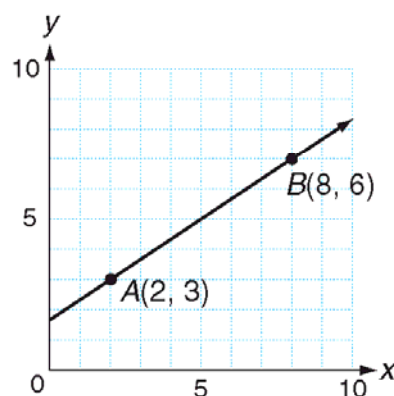
28. **Data Collection** Use a graphing calculator and a motion detector for the following. Set the equipment so that the graph shows distance on the y -axis and time on the x -axis.
- Experiment with walking in front of the motion detector. How must you walk to graph a straight line? Explain.
 - Describe what you must do differently to graph a line with a positive slope vs. a line with a negative slope.
 - How can you graph a line with slope 0? Explain.

EXPLORATION

5-4 The Slope Formula

In this Exploration, you will investigate a shortcut for finding the slope of a line.

1. What is the rise from point A to point B ?
2. What is the run from point A to point B ?
3. What is the slope of the line?
4. Explain how you could have found the rise by using only the y -coordinates of the two points, without seeing the graph.
5. Explain how you could have found the run by using only the x -coordinates of the two points, without seeing the graph.



THINK AND DISCUSS

6. **Describe** a general method you can use to find the slope of a line if you are given the coordinates of two points on the line.
7. **Explain** how you could use your method to find the slope of the line that contains the points $(5, 8)$ and $(10, 18)$.



Additional Examples

Example 1

Find the slope of the line that contains (2, 5) and (8, 1).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Use the .

$$= \frac{\square - \square}{\square - \square}$$

Substitute (,) for (x_1, y_1) and (,) for (x_2, y_2) .

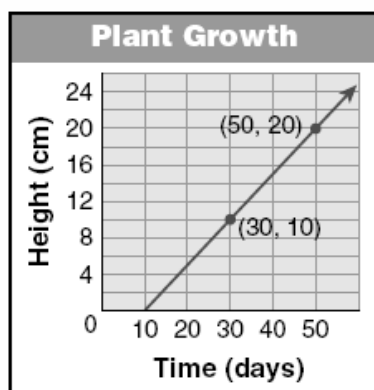
$$= \frac{\square}{\square}$$

Simplify.

$$= \square$$

The slope of the line containing (2, 5) and (8, 1) is .

3. The graph shows the height of a plant over a period of days. Find the slope of the line. Then tell what the slope represents.



In Lesson 5-3, slope was described as the constant rate of change of a line. You saw how to find the slope of a line by using its graph.

There is also a formula you can use to find the slope of a line, which is usually represented by the letter m . To use this formula, you need the coordinates of two different points on the line.



Slope Formula

WORDS	FORMULA	EXAMPLE
The slope of a line is the ratio of the difference in y -values to the difference in x -values between any two different points on the line.	If (x_1, y_1) and (x_2, y_2) are any two different points on a line, the slope of the line is $m = \frac{y_2 - y_1}{x_2 - x_1}$.	If $(2, -3)$ and $(1, 4)$ are two points on a line, the slope of the line is $m = \frac{4 - (-3)}{1 - 2} = \frac{7}{-1} = -7$.

EXAMPLE 1 Finding Slope by Using the Slope Formula



numbers from right tables are scripts. i "x sub y₂ as 2."

Find the slope of the line that contains $(4, -2)$ and $(-1, 2)$.

$$\begin{aligned}
 m &= \frac{y_2 - y_1}{x_2 - x_1} && \text{Use the slope formula.} \\
 &= \frac{2 - (-2)}{-1 - 4} && \text{Substitute } (4, -2) \text{ for } (x_1, y_1) \text{ and } (-1, 2) \text{ for } (x_2, y_2). \\
 &= \frac{4}{-5} && \text{Simplify.} \\
 &= -\frac{4}{5}
 \end{aligned}$$

The slope of the line that contains $(4, -2)$ and $(-1, 2)$ is $-\frac{4}{5}$.



- Find the slope of the line that contains $(-2, -2)$ and $(7, -2)$.
- Find the slope of the line that contains $(5, -7)$ and $(6, -4)$.
- Find the slope of the line that contains $(\frac{3}{4}, \frac{7}{5})$ and $(\frac{1}{4}, \frac{2}{5})$.

**MULTI-STEP
TEST PREP**

25. This problem will prepare you for the Multi-Step Test Prep on page 332.
- One way to estimate your maximum heart rate is to subtract your age from 220. Write a function to describe the relationship between maximum heart rate y and age x .
 - The graph of this function is a line. Find its slope. Then tell what the slope represents.

TEST PREP

26. The equation $2y + 3x = -6$ describes a line with what slope?
- (A) $\frac{3}{2}$ (B) 0 (C) $\frac{1}{2}$ (D) $-\frac{3}{2}$
27. A line with slope $-\frac{1}{3}$ could pass through which of the following pairs of points?
- (F) $(0, -\frac{1}{3})$ and $(1, 1)$ (H) $(0, 0)$ and $(-\frac{1}{3}, -\frac{1}{3})$
(G) $(-6, 5)$ and $(-3, 4)$ (J) $(5, -6)$ and $(4, 3)$
28. **Gridded Response** Find the slope of the line that contains $(-1, 2)$ and $(5, 5)$.

CHALLENGE AND EXTEND

Find the slope of the line that contains each pair of points.

29. $(a, 0)$ and $(0, b)$ 30. $(2x, y)$ and $(x, 3y)$ 31. (x, y) and $(x + 2, 3 - y)$

Find the value of x so that the points lie on a line with the given slope.

32. $(x, 2)$ and $(-5, 8)$, $m = -1$ 33. $(4, x)$ and $(6, 3x)$, $m = \frac{1}{2}$
34. $(1, -3)$ and $(3, x)$, $m = -1$ 35. $(-10, -4)$ and (x, x) , $m = \frac{1}{7}$
36. A line contains the point $(1, 2)$ and has a slope of $\frac{1}{2}$. Use the slope formula to find another point on this line.
37. The points $(-2, 4)$, $(0, 2)$, and $(3, x - 1)$ all lie on the same line. What is the value of x ? (*Hint: Remember that the slope of a line is constant for any two points on the line.*)

SPIRAL REVIEW

Solve each equation. Check your answer. (*Lesson 2-1*)

38. $k - 3.14 = 1.71$ 39. $-7 = p - 12$ 40. $25 = f - 16$
41. $-2 = 9 + n$ 42. $\frac{1}{5} + x = \frac{3}{5}$ 43. $a - \frac{1}{2} = \frac{3}{2}$

Tell whether the given ordered pairs satisfy a linear function. (*Lesson 5-1*)

44. $\{(1, 1), (2, 4), (3, 9), (4, 16)\}$ 45. $\{(9, 0), (8, -5), (5, -20), (3, -30)\}$

Use the intercepts to graph the line described by each equation. (*Lesson 5-2*)

46. $x - y = 5$ 47. $3x + y = 9$ 48. $y = 5x + 10$

MULTI-STEP TEST PREP



Characteristics of Linear Functions

Heart Health People who exercise need to be aware of their maximum heart rate.

- One way to estimate your maximum heart rate m is to subtract 85% of your age in years from 217. Create a table of values that shows the maximum heart rates for people ages 13 to 18. Then write an equation to describe the data in the table.
- Use your table from Problem 1 to graph the relationship between age and maximum heart rate. What are the intercepts? What is the slope?
- What do the intercepts represent in this situation?
- What does the slope represent? Explain why the slope is negative.
- Another formula for estimating maximum heart rate is $m = 206.3 - 0.711a$, where a represents age in years. Describe how this equation is different from your equation in Problem 1. Include slope and intercepts in your description.
- Which equation gives a higher maximum heart rate?
- To be exercising in your *aerobic training zone* means that your heart rate is 70% to 80% of your maximum heart rate. Write two equations that someone could use to estimate the range of heart rates that are within his or her aerobic training zone. Use your equation for maximum heart rate from Problem 1.



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