

Under Pressure: Inverse Relationship

Activity 15

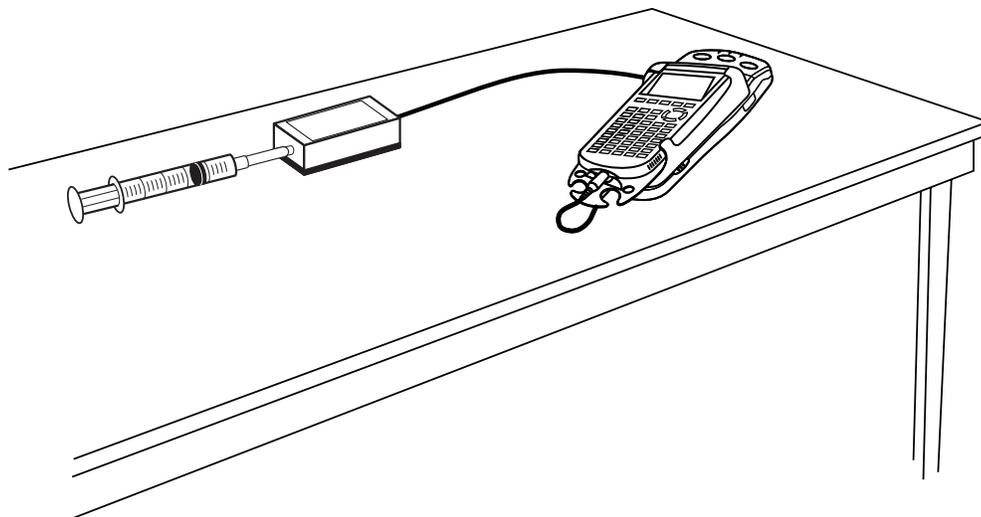
Let's take a sample of air in a closed container, and keep it at room temperature. If you change the volume of the container, what will happen to the air pressure inside? You can feel this by squeezing a small balloon in your hand. As the balloon gets smaller, you have to push harder. That is, as the volume decreases, the pressure increases. Two quantities that change this related way could be *inversely* related. If pressure and volume are inversely related, even if both quantities change, then their product stays the same.

Suppose that x and y represent the quantities that are inversely related. Then

$$xy = k, \text{ or } x = \frac{k}{y}$$

where k is a constant in both equations. Maybe you can think of some other quantities that also behave this way. For air and other gases, this relation has a name: *Boyle's law*.

In this activity, you will use a pressure sensor to investigate the relationship between pressure and volume for air contained within a closed syringe.



OBJECTIVES

- Record pressure versus volume data for a sample of air.
- Fit an inverse function model to the data.
- Use the table calculation feature of the TI handheld.
- Re-plot the data using linearization.

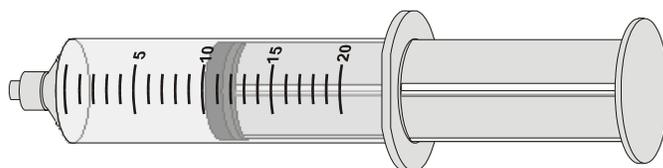
MATERIALS

TI-83 Plus, TI-83, or TI-73
CBL 2™ or LabPro® data collection device
DATAMATE software

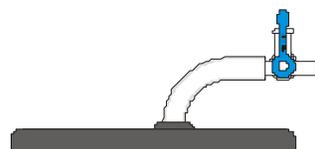
Vernier Gas Pressure Sensor or
Pressure Sensor with syringe
included

PROCEDURE

1. Prepare the Pressure Sensor and an air sample for data collection.
 - a. Connect the Pressure Sensor to CH 1 of the CBL 2 or LabPro. Use the unit-to-unit link cable to connect the TI Graphing Calculator to the data collection device. Firmly press in the cable ends.
 - b. With the syringe disconnected from the Pressure Sensor, move the piston of the syringe until the leading edge of the inside black ring is positioned at the 10.0 mL mark.



- c. Attach the syringe to the Pressure Sensor.
 - Newer Vernier Gas Pressure Sensors have a white stem protruding from the end of the sensor box. Attach the syringe directly to the white stem with a gentle half-turn.
 - Older Vernier Pressure Sensors have a 3-way valve at the end of a plastic tube leading from the sensor box. Before attaching the syringe, align the blue handle with the stem of the 3-way valve that will *not* have the tube connected to it, as shown in the figure at the right, closing this stem. Then attach the tube directly to the remaining open stem of the 3-way valve.



The air in the syringe/Pressure Sensor system is now sealed, so that as you move the piston, the air volume and pressure both change.

2. To obtain the best data possible, you will need to correct the volume readings from the syringe. Look at the syringe; its scale reports its own internal volume. However, that volume is not the total volume of trapped air in your system since there is a little bit of space in the tubing and internal to the pressure sensor.

To account for the extra volume in the system, you will need to add a small and constant volume to your syringe readings. If your sensor does not have a valve and tubing attached to it, the correction is 0.8 mL. If your sensor does have attached tubing, the correction is 1.5 mL. For example, with the 0.8 mL correction and a 5.0 mL syringe volume, the total volume would be 5.8 mL. *It is that total volume that you will need for the analysis.*

3. Turn on the calculator and start the DATAMATE software. Press **CLEAR** to reset the software.

4. Set up the calculator and data collection device for a Gas Pressure Sensor or Pressure Sensor.
 - a. Select 1: SETUP from the main screen.
 - b. If the calculator displays a Pressure Sensor with units of kPa in CH 1, proceed directly to Step 5. If it does not, continue with this step to set up the sensor manually.
 - c. Press **ENTER** to select CH 1.
 - d. Select PRESSURE from the SELECT SENSOR menu.
 - e. Select the correct pressure sensor (GAS PRESSURE SENSOR or PRESSURE SENSOR) from the PRESSURE menu.
 - f. Select the calibration listing for units of (KPA).
5. Set up the data-collection mode.
 - a. To select MODE, press **▲** once and press **ENTER**.
 - b. Select EVENTS WITH ENTRY from the SELECT MODE menu.
 - c. Select OK to return to the main screen.
6. You are now ready to collect pressure and volume data. It is best for one person to **handle** the gas syringe and for another to operate the calculator.
 - a. Select START to begin data collection.
 - b. Move the piston so the front edge of the inside black ring is positioned at the **5 mL** line on the syringe. Hold the piston firmly in this position until the pressure value displayed on the calculator screen stabilizes.
 - c. Press **ENTER**. (The person holding the syringe can relax after **ENTER** is pressed.) Type in 5, the gas volume (in mL) on the calculator. That is, enter either 5.8 or 6.5, as appropriate for your sensor. Remember, you are adding the constant volume to the apparent volume of the syringe. Press **ENTER** to store this pressure-volume data pair.
 - d. To collect another data pair, move the piston to 7.0 mL. When the pressure reading stabilizes, press **ENTER** and enter the total volume, remembering to add the appropriate correction to the syringe volume.
 - e. Continue with this procedure using volumes of 10.0, 12.0, 15.0, 17.0, and 20.0 mL.
 - f. Press **STO▶** when you have finished collecting data. A graph of pressure versus volume will be shown.
7. In order to work directly with the data, you need to leave the DATAMATE software. Press **ENTER** to return to the main screen, and select QUIT to leave the software. Follow any instructions on your screen to return to the calculator home screen.

ANALYSIS

1. Redisplay the graph.
 - a. Press **ZOOM**.
 - b. Press **▼** until ZoomStat is highlighted; press **ENTER** to display a graph with the x and y ranges set to fill the screen with data.
 - c. Press **TRACE** to determine the coordinates of a point on the graph using the cursor keys.

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2. Examine the data pairs on the displayed graph. As you move the cursor right or left, the volume (x) and pressure (y) values of each data point are displayed below the graph. Record the pressure (round to the nearest 0.1 kPa) and volume data values in the Data Table on the *Data Collection and Analysis* sheet.
3. To test Boyle's law you can plot the inverse function $y = k/x$ with your data. You will see this equation with the data, and can then adjust the value for k to improve the fit. First, enter the inverse function, and then in the next step, explore values of k .
 - a. Press $\boxed{Y=}$.
 - b. Press $\boxed{\text{CLEAR}}$ to remove any existing equation.
 - c. Enter k/x in the Y_1 field. (On the TI-73, access the alphabetic entry screen by pressing $\boxed{2\text{nd}} \boxed{\text{TEXT}}$.)
 - d. Press $\boxed{2\text{nd}} \boxed{\text{QUIT}}$ to return to the home screen.
4. Set a value for the parameter k , and then look at the resulting graph. To obtain a good fit, you will need to try several values. Use the steps below to store different values to the parameter k . Start with $k = 500$. Experiment until you find one that provides a good fit for the data.
 - a. Enter a value for the parameter k . Press $\boxed{\text{STO}} \boxed{\text{K}} \boxed{\text{ENTER}}$ to store the value in the variable K.
 - b. Press $\boxed{\text{GRAPH}}$ to see your data with the model graph superimposed.
 - c. Press $\boxed{2\text{nd}} \boxed{\text{QUIT}}$ to return to the home screen.

⇒ Answer Question 1 on the *Data Collection and Analysis* sheet.

5. Find the products of the data coordinates and record them in the third column of the Data Table.

⇒ Answer Question 2 on the *Data Collection and Analysis* sheet.

6. The modeling equation you have determined in this activity can be used to predict syringe pressure values for given volumes. The second Data Table on the *Data Collection and Analysis* sheet lists a number of volumes. Fill in the corresponding pressures. Although it would not take long to do this by hand, try using the table feature of your calculator to do the calculation.
 - a. Press $\boxed{2\text{nd}} \boxed{\text{TBLSET}}$ to display Table Setup.
 - b. Use the cursor keys to highlight Ask on the Indpnt: line, and press $\boxed{\text{ENTER}}$ to select this mode.
 - c. Use the cursor keys to highlight Auto on the Depend: line, and press $\boxed{\text{ENTER}}$ to select this mode.
 - d. Press $\boxed{2\text{nd}} \boxed{\text{TABLE}}$ to display the table.
 - e. Enter 2.5 for the first x value, and finish the entry with $\boxed{\text{ENTER}}$. Note how the calculator has determined the y value from the model. Complete the Data Table for the remaining three entries.

⇒ Answer Questions 3 and 4 on the *Data Collection and Analysis* sheet.

7. Another way to see if two quantities are inversely related is to plot the one quantity versus the inverse of the other. For this pressure activity, that would be pressure versus the inverse of the volume. If the new plot shows a line of proportionality, then the original data is inversely related. You can see this by noting the simple rearrangement of the equation $y = k/x$ as $y = k \times (1/x)$. Written this way, you can see that y is proportional to the quantity $(1/x)$. The process of graphing calculated quantities to obtain a straight-line graph is sometimes called *linearization*.

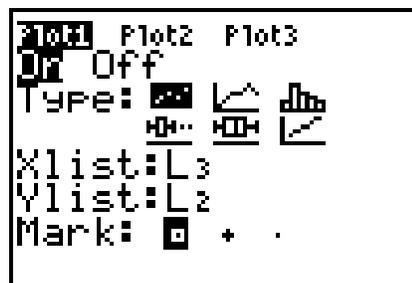
To see if the data is inversely related using this test, you need to define a new column containing the inverse of the volume data. The volume data is in L1; L3 is available for the inverse volume information. L2 will continue to hold the pressure data.

TI-73

- Press $\boxed{2nd}$ [QUIT] to return to the home screen.
- Press $1/L1$ \boxed{STO} \blacktriangleright $L3$ \boxed{ENTER} . To enter list names on the TI-73, press $\boxed{2nd}$ [STAT] to see a menu of list names. L3 will now hold the inverse of the values in L1.

TI-83, TI-83 Plus

- Press $\boxed{2nd}$ [QUIT] to return to the home screen.
 - Press $1/$ $\boxed{2nd}$ [L1] \boxed{STO} \blacktriangleright $\boxed{2nd}$ [L3] \boxed{ENTER} . L3 will now hold the inverse of the values in L1.
8. You can plot the pressure versus inverse volume data on a new graph using the following commands.
- Press $\boxed{2nd}$ [STAT PLOT] ([PLOT] on the TI-73) and press \boxed{ENTER} to select Plot 1.
 - Change the Plot1 settings to match the screen shown here. Press \boxed{ENTER} to select any of the settings you change.
 - Press \boxed{ZOOM} and then select ZoomStat (use cursor keys to scroll to ZoomStat) to draw a graph with the x and y ranges set to fill the screen with data.



⇒ Answer Question 5 on the *Data Collection and Analysis* sheet.

EXTENSIONS

1. An alternate way to find a model for the data you collected involves a process known as *regression analysis*. The calculator will select the best parameters for the power function $y = ax^b$. (The TI-73 cannot perform this analysis.)

Note: Before performing this extension, the Stat Plot 1 settings must first be restored to graph L1 and L2 using the plotting steps of Step 8, with L1 replacing L3.

- a. Press **STAT** and use the cursor keys to highlight CALC.
- b. Press the number adjacent to PwrReg to copy the command to the home screen.
- c. Press **2nd** [L1] **,** **2nd** [L2] **,** to enter the lists containing your data.
- d. Press **VARS** and use the cursor keys to highlight Y-VARS.
- e. Select Function by pressing **ENTER**.
- f. Press **ENTER** to copy Y1 to the home screen.
- g. On the home screen, you will now see the entry PwrReg L1, L2, Y1. This command will perform a power regression with L1 as the x and L2 as the y values. The resulting regression line will be stored in equation variable Y1. Press **ENTER** to perform the regression.
- h. Press **ZOOM** and then select ZoomStat (use cursor keys to scroll to ZoomStat) to draw a graph with the x and y ranges set to fill the screen with data.

Does the power law model provide a good fit to your pressure versus volume data? How many parameters are adjusted to achieve a good fit compared to the inverse model?

2. It is possible to deduce the internal volume of the sensor rather than using the supplied values. If you assume Boyle's law, you can determine the internal volume from the y -intercept value. For example, if the internal volume of the sensor is d , then the total volume of air in the system is $V + d$. From Boyle's law, we have $P*(V + d) = k$, which can be rearranged to $V = k*(1/P) - d$. From the form of this equation, we can see that the y -intercept of a V (y -axis) versus $1/P$ (x -axis) graph is the negative of the internal volume.

Note that this is not the same graph setup as used in the activity. Collect a new data set, but do not correct the volumes; just enter the syringe volume directly. Create a graph of V versus $1/P$. Fit a straight line to the data, and from the y -intercept determine the internal volume of your particular sensor. For comparison to your results, the internal volume of the Vernier Gas Pressure Sensor (model without attached tubing and valve) is about 0.8 mL, while for the Vernier Pressure Sensor (with tubing and valve) the aggregate internal volume is about 1.5 mL.

DATA COLLECTION AND ANALYSIS

Name _____

Date _____

DATA TABLES

Volume in mL (x values)	Pressure in kPa (y values)	Product (x*y)

Volume in mL (x values)	Pressure in kPa (y values)
2.5	
17.8	
520	
0.0012	

QUESTIONS

1. In the space below, use the value for k to record the model equation that gives the best fit to the data.

