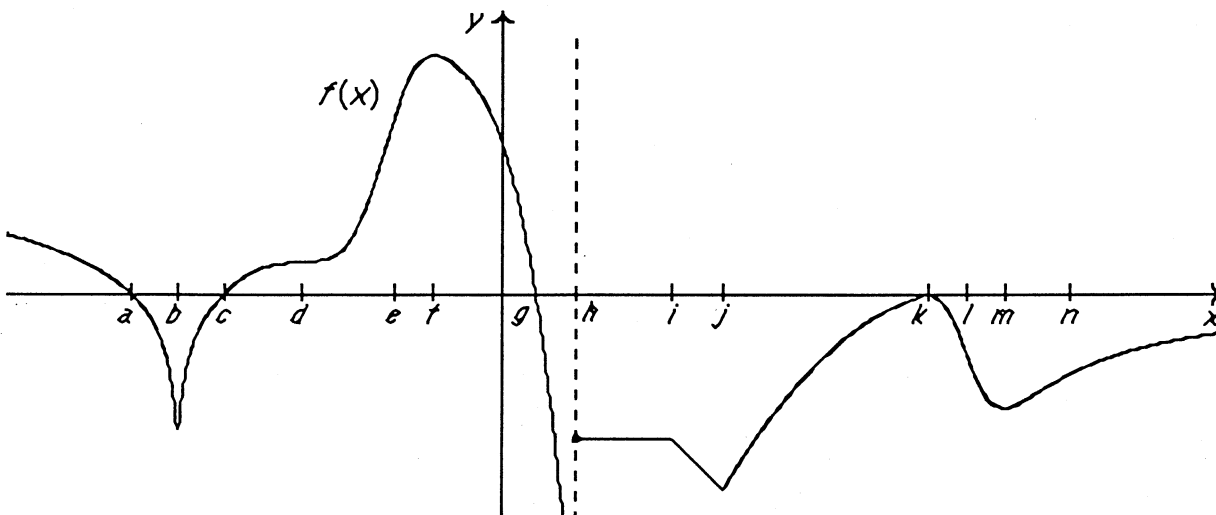


Concepts Worksheet 6

Chapter 3 For use after Article 3.4.

An Unusual Function



1. The function f drawn above would be difficult to describe algebraically; nevertheless, it has interesting geometric features for which calculus provides descriptions. Using the textbook definitions and some freedom of artistic judgment, name the x coordinate(s) for:

a) roots of $f(x) = 0$ _____

b) points of discontinuity of f _____

c) critical points _____

d) intervals over which f increases _____

e) intervals over which f decreases _____

f) relative maxima _____

g) absolute maxima _____

h) relative minima _____

i) absolute minima _____

j) intervals over which f is concave up _____

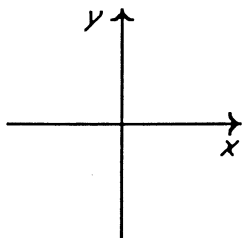
k) intervals over which f is concave down _____

l) points of inflection _____

m) equation of horizontal asymptote(s) _____

n) equation of vertical asymptote(s) _____

b) Note that $x^5 + x^4y - xy^2 - y^3 = (x + y)(x^2 + y)(x^2 - y) = 0$. Use this factorization to graph the function $y = f(x)$, described earlier.



c) Evaluate $\lim_{x \rightarrow 0^+} \frac{f(x) - f(0)}{x - 0} =$ _____

d) Evaluate $\lim_{x \rightarrow 0^-} \frac{f(x) - f(0)}{x - 0} =$ _____

e) $f'(0) =$ _____ Why? _____

Concept Connectors

3. If $f(g(x)) = g(f(x)) = x$, what is the relationship between functions f and g ? _____

4. Find an expression for $g'(x)$ using $f(g(x)) = x$, assuming both f and g are differentiable. _____

5. Let f and its inverse, f^{-1} , be differentiable functions with $f(x)$ values at $x = 1$, $x = 2$, and $x = 3$ as indicated in the table below:

x	$f(x)$	$f'(x)$
1	3	4
2	1	5
3	2	6

HINT:

a) Point on f \Rightarrow Point on f^{-1}
 $(1, 3) \Rightarrow (3, 1)$
 $(2, 1) \Rightarrow (\text{---}, \text{---})$
 $(3, 2) \Rightarrow (\text{---}, \text{---})$

b) Find the derivative of f^{-1} at $\begin{cases} x = 1 \text{ ---} \\ x = 2 \text{ ---} \end{cases}$