Exploring Continuity

In previous math courses, you have used a "touchy-feely" definition of continuity. *A function is continuous if you can trace its graph without picking up your pencil.*. Now that you are in Calculus, let's look at a more formal definition of continuity, first in English and then in the language of mathematics.

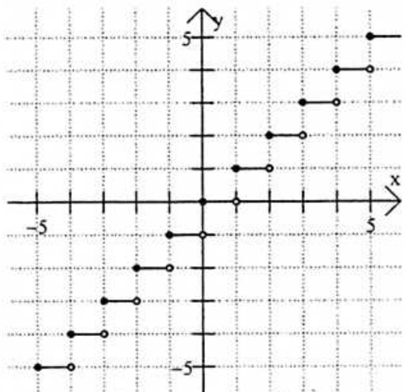
In English:
A function is continuous at a point if the limit of the function exists at that point and equals the value of the function (y-value) at that point.

In Math:
Interior Point: A function y = f(x) is continuous at an interior point *c* of its domain if $\lim_{x \to c} f(x) = f(c)$.

Endpoint: A function y = f(x) is continuous at a left endpoint *a* or is continuous at a right endpoint *b* of its domain if $\lim_{x \to a^+} f(x) = f(a)$ or $\lim_{x \to b^-} f(x) = f(b)$ respectively.

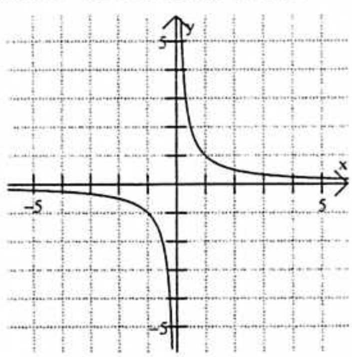
Examples:

1. $f(x) = \text{int}\, x$



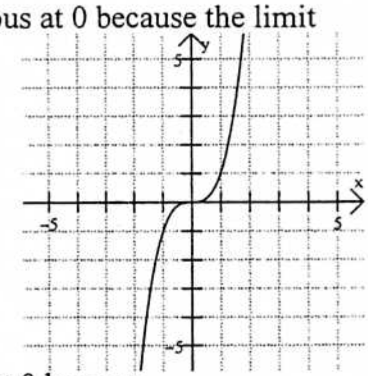
   a. f(0) = 0
   b. $\lim_{x \to 0^-} f(x) = -1$
   c. $\lim_{x \to 0^+} f(x) = 0$
   d. $\lim_{x \to 0} f(x) = DNE$
   e. f(x) is **not** continuous at 0 because the limit does not exist.

2. $f(x) = \dfrac{1}{x}$



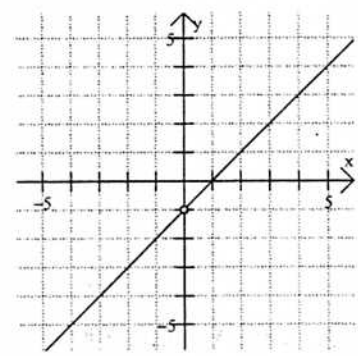
   a. f(0) is undefined
   b. $\lim_{x \to 0^-} f(x) = -\infty$
   c. $\lim_{x \to 0^+} f(x) = \infty$
   d. $\lim_{x \to 0} f(x) = DNE$
   e. f(x) is **not** continuous at 0 because the limit does not exist.

3. $f(x) = x^3$



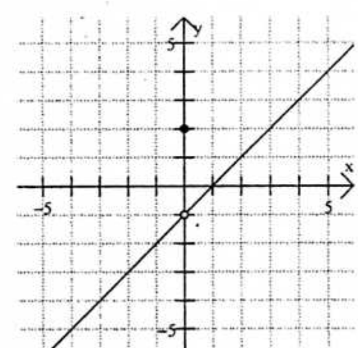
   a. f(0) = 0
   b. $\lim_{x \to 0^-} f(x) = 0$
   c. $\lim_{x \to 0^+} f(x) = 0$
   d. $\lim_{x \to 0} f(x) = 0$
   e. f(x) **is** continuous at 0 because $\lim_{x \to 0} f(x) = f(0)$.

4. $f(x) = \dfrac{x^2 - x}{x}$



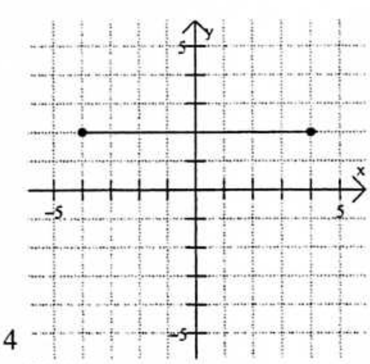
   a. f(0) is undefined
   b. $\lim_{x \to 0^-} f(x) = -1$
   c. $\lim_{x \to 0^+} f(x) = -1$
   d. $\lim_{x \to 0} f(x) = -1$
   e. f(x) is **not** continuous at 0 because f(0) is undefined

5. $f(x) = \begin{cases} x - 1, x \neq 0 \\ 2, x = 0 \end{cases}$



   a. f(0) = 2
   b. $\lim_{x \to 0^-} f(x) = -1$
   c. $\lim_{x \to 0^+} f(x) = -1$
   d. $\lim_{x \to 0} f(x) = -1$
   e. f(x) is **not** continuous at 0 because $\lim_{x \to 0} f(x) \neq f(0)$.

6. $f(x) = 2, -4 \leq x \leq 4$



   a. f(4) = 2
   b. $\lim_{x \to 4^-} f(x) = 2$
   c. f(-4) = 2
   d. $\lim_{x \to -4^+} f(x) = 2$
   e. f(x) **is** continuous at 4 because $\lim_{x \to 4^-} f(x) = f(4)$.
   f. f(x) **is** continuous at -4 because $\lim_{x \to -4^+} f(x) = f(-4)$.

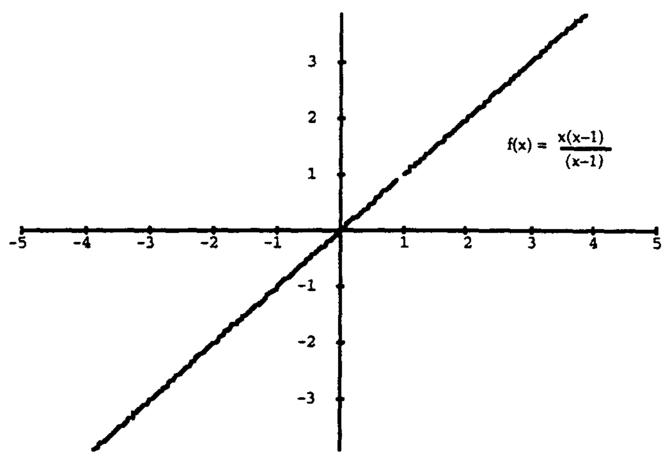
3.2 Continuity

A function f is **continuous at a point a** provided $\lim_{x \to a} f(x)$ exists and, moreover, that the value of the limit is f(a); that is $\lim_{x \to a} f(x) = f(a)$. If a function is <u>not</u> continuous at a, we say f is **discontinuous at a**, or f has a **discontinuity at a.**

Continuity is a point property of a function. However, if a function is continuous at each point in its domain it is called a continuous function. Geometrically we consider a function continuous if its graph contains no breaks, jumps or wild oscillations.

Example: Investigate the continuity of the function $f(x) = \dfrac{x(x-1)}{x-1}$ on the interval [–3,3].

<u>Solution</u>: Enter and graph the function f in the ZDecimal viewing rectangle. From the graph of f it appears that f is defined and continuous everywhere except at x = 1, where f is undefined and therefore discontinuous.

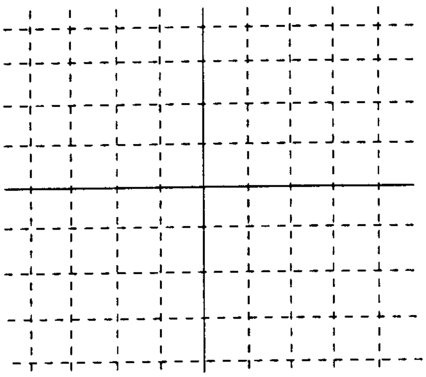


Since the screen resolution of the TI-82 is low, discontinuities or breaks in the function graph may not show up. Thus to determine conclusively whether a function is continuous we must use our knowledge of its domain and limits.

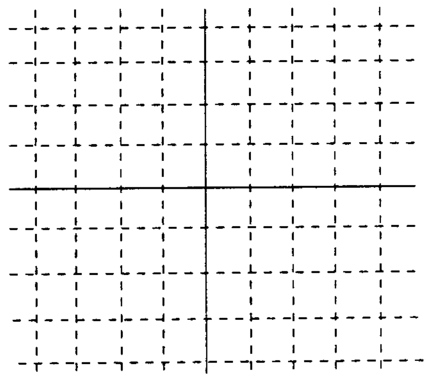
3.2 Exercises

Using the ZDecimal viewing rectangle, investigate the continuity of each function on the interval [–4,4].

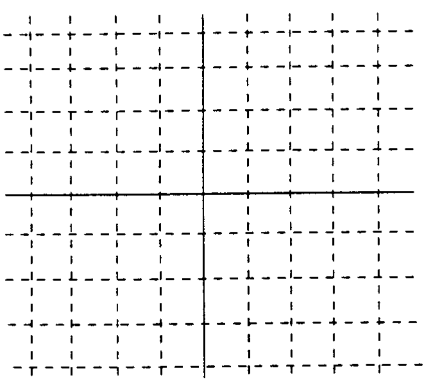
1. $f(x) = \dfrac{1}{1-x^2}$



2. $f(x) = \tan x$



3. $f(x) = \dfrac{x}{x^2+2x+2}$



4. $f(x) = \dfrac{1-\cos x}{x+x^2}$