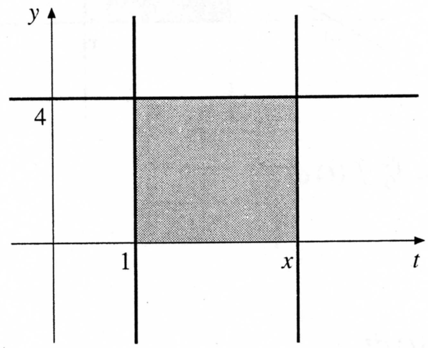
GROUP WORK 1, SECTION 5.2

The Area Function

Recall that we can use the notation $\int_a^b f(t)\,dt$ to denote the area under the curve $f(t)$ between $t = a$ and $t = b$.

1. Consider the constant function $f(t) = 4$.



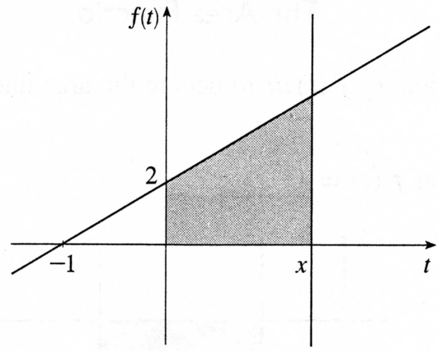
(a) Using geometry, compute $\int_1^2 f(t)\,dt$.

(b) Similarly compute $\int_1^3 f(t)\,dt$ and $\int_1^4 f(t)\,dt$.

(c) Using your answers to parts (a) and (b) as a guide, compute $\int_1^x f(t)\,dt$ for any $x \geq 1$.

(d) We now define the *area function* $A(x) = \int_1^x f(t)\,dt$, $1 \leq x \leq 4$. What is $A(2)$? $A(2.5)$? $A(1)$? Write a general formula for $A(x)$.

2. Let $f(t) = 2t + 2$ for all t.



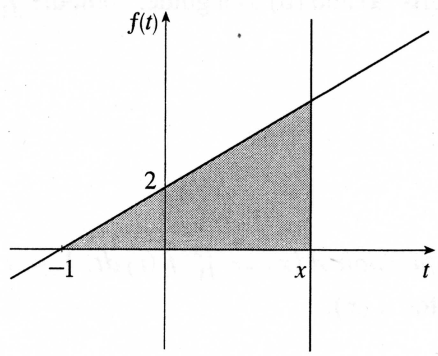
(a) Using geometry, compute $\int_0^2 f(t)\,dt$.

(b) Similarly, compute $\int_0^4 f(t)\,dt$.

(c) Using your answers to parts (a) and (b) as a guide, compute $\int_0^x f(t)\,dt$ for any $x \geq 0$.

(d) We now define another area function $B(x) = \int_0^x f(t)\,dt$. What is $B(2)$? $B(4)$? $B(0)$? Write a general formula for $B(x)$.

(e) We will now define a third area function $C(x) = \int_{-1}^x f(t)\,dt$ for any $x \geq -1$, as pictured below:



What is $C(2)$? $C(4)$? $C(-1)$? Write a general formula for $C(x)$.

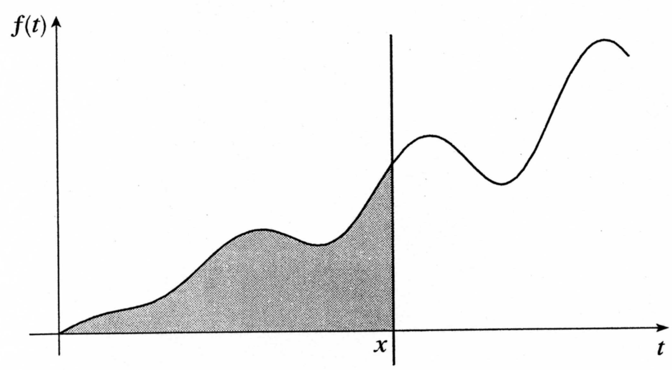
3. The Punchline:

We have now computed three different area functions. Fill in the blanks:

$f(t) = 4$	$A(x) =$	$A'(x) =$
$f(t) = 2t + 2$	$B(x) =$	$B'(x) =$
$f(t) = 2t + 2$	$C(x) =$	$C'(x) =$

You should notice a very interesting fact about the derivatives of the area functions — a fundamentally beautiful property. What is it?

4. We are going to define one final function, $D(x) = \int_0^x 0.2t \sin(\cos(\sin t))\,dt$.



Don't worry about trying to find a simple formula for $D(x)$. But, using our amazing fact, fill in the last blank:

$D'(x) =$

Not only is the fact that we've discovered a surprising one; as we shall see in Section 5.4, it is also extremely useful.