GROUP WORK 3, SECTION 2.2

Why Can't We Just Trust the Table?

We are going to investigate $\lim_{x \to 0} \sin \frac{\pi}{x}$. We will take values of x closer and closer to zero, and see what v

the function approaches.

1. Your teacher has given you a digit — let's call it d. Fill out the following table. If, for example, digit is 3, then you would compute $\sin\left(\frac{\pi}{0.3}\right)$, $\sin\left(\frac{\pi}{0.03}\right)$, $\sin\left(\frac{\pi}{0.003}\right)$, $\sin\left(\frac{\pi}{0.0003}\right)$, etc.

x	$\sin \frac{\pi}{x}$
$0.d$	
$0.0d$	
$0.00d$	
$0.000d$	
$0.0000d$	
$0.00000d$	

2. What is $\lim_{x \to 0} \sin \frac{\pi}{x}$?

3. Now fill out the table with a different digit.

x	$\sin \frac{\pi}{x}$
$0.d$	
$0.0d$	
$0.00d$	
$0.000d$	
$0.0000d$	
$0.00000d$	

Do you get the same result?

4. What is $\lim_{x \to 0} \sin \frac{\pi}{x}$?

GROUP WORK 2, SECTION 2.3

Fixing a Hole

Consider $f(x) = \dfrac{x-2}{x^2-x-2}$.

1. Is $f(x)$ defined for $x = -1$? For $x = 0$? For $x = 1$? For $x = 2$?

2. What is the domain of f?

3. Compute $\lim_{x \to -1} \dfrac{x-2}{x^2-x-2}$ and $\lim_{x \to 2} \dfrac{x-2}{x^2-x-2}$. Notice that one limit exists, and one does not.

4. Graph $y = \dfrac{x-2}{x^2-x-2}$. There are two x-values that are not in the domain of f. Later, we will call these "discontinuities." Geometrically, what is the difference between the two discontinuities?

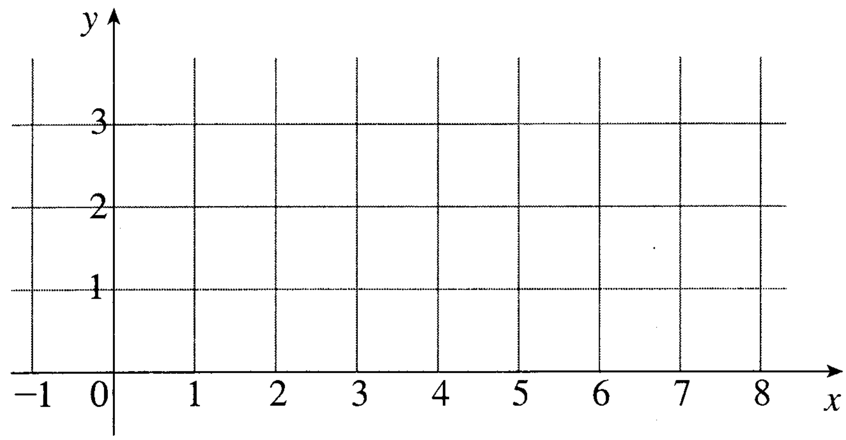
5. We say that $f(x)$ has one *hole* in it. Where do you think that the hole is? Define "hole" in this context.

6. The function $g(x) = \dfrac{\sin x}{x}$ is not defined at $x = 0$. Sketch this function. Does it have a hole at $x = 0$?

What's the Pattern?

Consider the function $f(x) = \sqrt{1+x}$.

1. Carefully sketch a graph of this function on the grid below.



2. Sketch the secant line to f between the points with x-coordinates $x = 2$ and $x = 4$.

3. Sketch the secant lines to f between the pairs of points with the following x-coordinates, and compute their slopes:

(a) $x = 2$ and $x = 3$ (b) $x = 3$ and $x = 4$ (c) $x = 2.5$ and $x = 3.5$ (d) $x = 2.8$ and $x = 3.2$

4. Using the slopes you've found so far, estimate the slope of the tangent line at $x = 3$.

5. Repeat Problem 4 for $x = 8$.

6. Based on Problems 4 and 5, guess the slope of the tangent line at any point $x = a$, for $a > -1$.

Slope Patterns

1. (a) Estimate the slope of the line tangent to the curve $y = 0.1x^2$, where $x = 0, 1, 2, 3$. Use your information to fill in the following table:

x	slope of tangent line
0	
1	
2	
3	

(b) You should notice a pattern in the above table. Using this pattern, estimate the slope of the line tangent to $y = 0.1x^2$ at the point $x = 57.5$.

2. Consider the function $f(x) = 0.1x^3 - 3x$.

(a) On what intervals is this function increasing? On what intervals is it decreasing?

(b) On what interval or intervals is the slope of the tangent line positive? On what interval or intervals is the slope of the tangent line negative? What is the connection between these questions and part (a)?

(c) Where does the slope of the tangent line appear to be zero? What properties of the graph occur at these points?

(d) Where does the tangent line appear to approximate the curve the best? The worst? What properties of the graph seem to make it so?