Mini–Lab 23 Increasing/Decreasing Functions

Purpose

- To investigate a common quadratic model.
- To determine intervals on which a given model is increasing and intervals on which a given model is decreasing.
- To introduce variable rates of change.

Background

If an earthly object is projected straight up into the air, then the height h (in feet) of the object above the earth is given by a quadratic function of the time t (in seconds) elapsed since the object was shot or thrown into the air. The general model is described by the equation

$$h(t) = -16t^2 + v_0 t + h_0$$

where v_0 is the initial velocity (in feet/second) and h_0 is the initial height (in feet) above the earth.

Investigations

Problem Situation: A small rocket is fired vertically into the air with an initial velocity of 125 feet/second. The launchpad of the rocket is 8 feet above the ground when the rocket is fired. After reaching some maximum height, the rocket turns downward. Let h feet represent the height of the rocket after t seconds.

1. Write the algebraic model for the problem situation. Enter this model into your calculator.
2. Use **TABLE** to complete Table 1.

Time (seconds)	Height (feet)
0	
1	
2	
3	
4	
5	
6	
7	
8	

Table 1: Rocket's time versus height

3. Graph the function and use **maximum** under the **CALC** menu to approximate the maximum height that the rocket attains. How long will it take to reach this height? Write a complete sentence describing your answers.

4. Graph the function and use **root** under the **CALC** menu to approximate the time when the rocket hits the ground. Write a complete sentence describing your answer.

5. Refer to your answers to investigations 3 and 4. On what interval(s) are the function values increasing? On what interval(s) are the function values decreasing? Interpret your answers in terms of the motion of the rocket.

6. For each one–second interval from Table 1, calculate the change in height (the first finite difference in outputs). There are several ways to do this on the calculator. We'll look at two techniques. Be sure the function is entered as Y_1.

TI–82 Procedure to find successive changes in output one at a time

- In the home screen, Type $\mathbf{Y_1(1) - Y_1(0)}$ followed by ENTER.

The answer is the change in height during the first second of flight.

- Press 2nd ENTER to recall the previous command. Edit the command to read $\mathbf{Y_1(2) - Y_1(1)}$. Press ENTER.

The answer is the change in height during the 2nd second of flight. This procedure can be repeated until you have $\mathbf{Y_1(8) - Y_1(7)}$.

Another approach is to generate the list of changes all at once. Not surprisingly, this is called a list on the TI–82. We will use a new command, the **seq** command, to generate this list. The word **seq** is short for sequence.

The general syntax for **seq** is

seq(formula, variable, initial value of variable, final value of variable, increment).

In our case, the formula is $\mathbf{Y_1(X + 1) - Y_1(X)}$, the variable is **X**, the initial value of X is **1**, the final value of X is **7**, and the increment is **1**.

TI–82 Procedure to use seq to calculate finite differences

- In the home screen press 2nd STAT to open the **LIST** menu.
- Type **5** to select **seq(**.
- Type $\mathbf{Y_1(X + 1) - Y_1(X), X, 0, 7, 1)}$ followed by ENTER.

A list of values representing the successive changes in height is displayed. You can use the right and left arrow keys to scroll through the list. Record this information in Table 2.

Beginning and ending times (seconds)	Change in time (seconds)	Change in height (feet)
0 to 1	1	
1 to 2	1	
2 to 3	1	
3 to 4	1	
4 to 5	1	
5 to 6	1	
6 to 7	1	
7 to 8	1	

Table 2: Rate of change in height per unit change in time

7. Based on Table 2, on what time interval(s) is the height changing the fastest? On what time interval(s) is the height changing the slowest? Write a complete sentence describing your answers in terms of the problem situation.

8. a. What physical quantity is represented by the ratio of the change in height to the change time?

   b. Is this quantity constant in this problem? Defend your answer.

9. Explain what it means in terms of the rocket when the ratio of the change in height to the change in time is

   a. positive

   b. negative

   c. near zero

   d. very large in magnitude.

10. Approximately how fast is the rocket travelling when it hits the ground? Justify your answe