Mini–Lab 27   Factor Theorem

Purpose

To search for a relationship between the factors, the real number zeros, and the x–intercepts of a polynomial function.

Investigations

1. Given $f(x) = (x-2)(x+3)$:

   a. What is the degree of $f(x)$? _____

   b. List each factor of $f(x)$.

   c. Display a table of outputs. List all the real number inputs where the outputs of the function are zero. These are the real number zeros of the function.

   d. Graph the function in the viewing window Xmin = –9.4, Xmax = 9.4, Ymin = –10, Ymax = 10. Record the x–intercepts.

   e. How are the real number zeros and the x–intercepts related?

2. Given $g(x) = (x-1)(x-5)(x+2)(x+7)$:

   a. What is the degree of $g(x)$? _____

   b. List each factor of $g(x)$.

   c. Predict the real number zeros of $g(x)$. Give a reason for your prediction. Display a table of outputs. Identify the real number zeros of $g(x)$ from the table. If these differ from your prediction, modify your explanation to account for the differences.

d. Predict the x–intercepts of $g(x)$. Give a reason for your prediction. Graph the function in the viewing window Xmin = –9.4, Xmax = 9.4, Ymin = –400, Ymax = 300. Record the x–intercepts. If these differ from your prediction, modify your explanation to account for the differences.

e. If you know the factors of a polynomial, how would you find the real number zeros?

3. Given $h(x) = (x+8)(x-3)(x-5)(x+4)(x+6)$:

a. What is the degree of $h(x)$? _____

b. List each factor of $h(x)$.

c. Predict the real number zeros of $h(x)$. Give a reason for your prediction. Display a table of outputs. Identify the real number zeros of $h(x)$ from the table. If these differ from your prediction, modify your explanation to account for the differences.

d. Predict the x–intercepts of $h(x)$. Give a reason for your prediction. Graph the function in the viewing window Xmin = –9.4, Xmax = 9.4, Ymin = –2000, Ymax = 3000. Record the x–intercepts. If these differ from your prediction, modify your explanation to account for the differences.

e. If you know the real number zeros of a polynomial, how can you predict the x–intercepts?

4. Given $F(x) = (x-7)(x-4)(x+3)(x+7)(x+5)(x-1)(x-9)$:

   a. What is the degree of $F(x)$? _____

   b. List each factor of $F(x)$.

   c. Predict the real number zeros of $F(x)$. Give a reason for your prediction. Display a table
       of outputs. Identify the real number zeros of $F(x)$ from the table. If these differ from your
       prediction, modify your explanation to account for the differences.

   d. Predict the x–intercepts of $F(x)$. Give a reason for your prediction. Graph the function in
       the viewing window Xmin = –9.4, Xmax = 9.4, Ymin = –70000, Ymax = 70000. Record
       the x–intercepts. If these differ from your prediction, modify your explanation to account
       for the differences.

   e. If you know a real number zero of a function, how would you write a factor of the func-
       tion?

Use investigations 1–4 to help you answer the following questions. Assume that a is a real num-
ber.

5. If $x - a$ is a factor of function $f(x)$, what is a real number zero of $f(x)$?_____

6. If a is an x–intercept of the graph of $y = f(x)$, then $f(a) =$ _____?

7. If a is an x–intercept of the graph of $y = f(x)$, then what is a factor of $f(x)$?

8. The graph of $y = f(x)$ has x–intercepts at 3, 5, and –2, and nowhere else.

   a. What are the factors of $f(x)$?

   b. If $f(x)$ is a polynomial function of degree three, write an algebraic expression for $f(x)$. Check your answer using a graph.