Mini–Lab 28 Real Number Zeros of Polynomials

Purpose

To find exact values for all the real number zeros of a polynomial.

Investigations

Given the function $f(x) = 5x^4 - 6x^3 - 211x^2 + 510x - 154$, find exact values for the real number zeros of $f(x)$.

1. Make a list of the possible rational number solutions. To do so:

   a. List all the factors of 154, which is the absolute value of the constant term.

   b. List all the factors of 5, which is the leading coefficient.

   c. Create all possible fractions using answers to investigation 1a for the numerator and answers to investigation 1b as the denominator. These are the possible rational number zeros of $f(x)$.

2. Enter the function $f(x)$ as Y_1. Graph in the viewing window Xmin = –9.4, Xmax = 9.4, Ymin = –2000, Ymax = 1000. Use the graph to answer the following questions.

   a. How many real number zeros does $f(x)$ have? Why?

   b. Use **TRACE** to list the approximate values for the zeros of $f(x)$. Do not use zoom.

   c. Are any of the zeros you listed exact? Which ones? Why?

   d. If any of your zeros are exact, they must be in the list of rational number solutions (investigation 1c). Verify this fact.

3. There is one integer zero (investigation 2c). **Record the integer zero:** ____________

4. You will use **synthetic division** before determining the next zero. If the zero written in investigation 3 is a, then divide $f(x)$ by $x - a$. You need the quotient of this division.

   a. Do the synthetic division by hand below. Write the quotient and the remainder.

   b. How can you be sure you did the synthetic division correctly?

5. The synthetic division can also be done on the calculator. Follow the procedure and compare to your work in investigation 4a.

TI–82 Procedure for synthetic division

- Store the zero listed in investigation 3 in location x.
- In the home screen, enter **5** and press ENTER. The 5 is the leading coefficient of $f(x)$. The 5 also is the leading coefficient of the quotient.
- Press the **multiplication key** followed by X, T, θ + (–6). **Note:** –6 is the next coefficient in $f(x)$. The answer is the second coefficient in the quotient.
- Press the 2nd ENTER to create an editable version of the last entry. Change the –6 to **–211**, the next coefficient in $f(x)$. Press ENTER. The third coefficient in the quotient is output.
- Press the 2nd ENTER to create an editable version of the last entry. Change the –211 to **510**, the next coefficient in $f(x)$. Press ENTER and the fourth coefficient in the quotient is output.
- Press the 2nd ENTER to create an editable version of the last entry. Change the 510 to **–154**, the last coefficient in $f(x)$. Press ENTER and the remainder is output. If all has gone well, this should be zero.

Record the quotient:________________________________

6. There is one non–integer, rational number zero (investigation 2c)
   Record this zero: ____________

7. Use synthetic division on the quotient function from investigation 5. Let x be the zero listed in investigation 6. The resulting quotient function should be quadratic.

   Record the quotient: ______________________________

8. The are two irrational zeros. These are found by setting the quadratic function from investigation 7 equal to zero and solving the resulting equation. Find these two zeros below. Show all work.

9. It is easy to make a mistake finding the irrational zeros exactly. They should be checked by substituting them into the original function. This can be done using the **Ask** feature in the table. It can also be done in the home screen.

TI–82 Procedure to check irrational zeros

- Type Y_1(exact irrational zero) followed by ENTER.
- Press 2nd ENTER to recall the previous command. Edit this by changing the input to the other irrational zero. This should require a sign change only.
- Press ENTER.

Are your irrational zeros correct? How do you know? If not, revise investigation 8 and redo this investigation.

10. You now know all four zeros of the original function $f(x)$.

   a. Record them here:

   b. If we know all the zeros of a polynomial function, we can write the function in factored form by using the Factor Theorem. One reminder: If the fraction $\frac{a}{b}$ is a zero of a polynomial, then the corresponding factor is $bx - a$.

   Use these zeros to write the function $f(x)$ in factored form.

   c. Enter the factored form of $f(x)$ as Y_2. Look at a table of outputs for Y_1 and Y_2. Is your factorization correct? How can you tell? If not, repeat investigation 10b and use the table to check your answer.

11. Based on this mini–lab, describe, in detail, a procedure for finding the exact zeros of any polynomial function.