That's the way the Ball Bounces

As the ball bounces up and down, the maximum height it reaches continually decreases from one bounce to the next. For any particular bounce, if the ball's height is plotted as a function of time, the resulting graph has a parabolic shape. The relationship between height and time for a single bounce of a ball, then, is quadratic. Expressed mathematically:

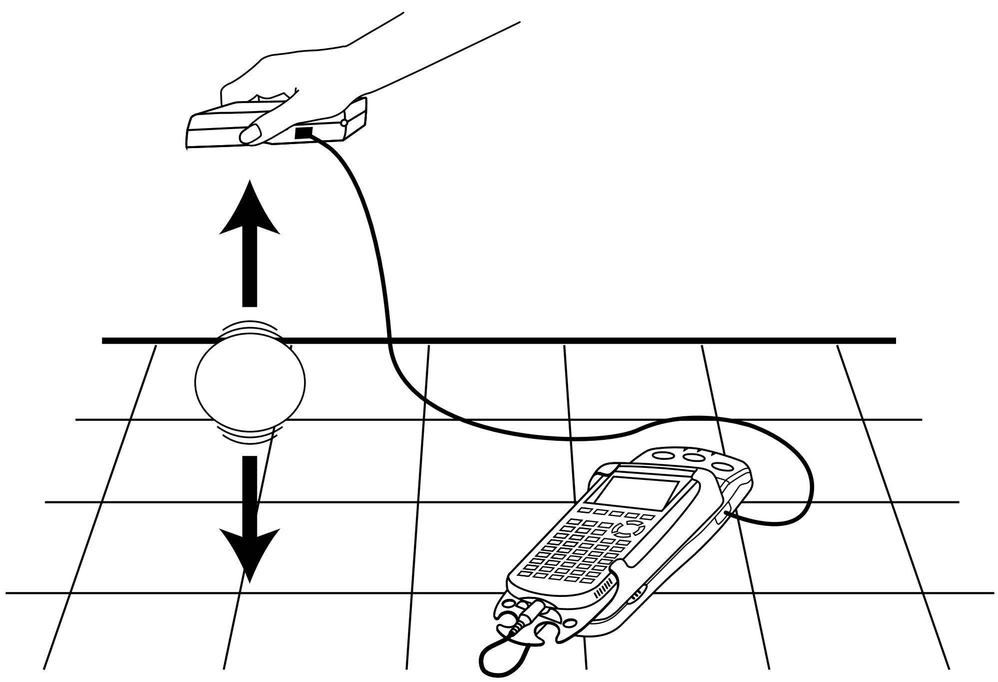
$$y = ax^2 + bx + c$$

where y represents the ball's height at any given time, x. It is possible to mathematically model a ball's bouncing behavior using a series of quadratic functions.

In this activity, you will record the motion of a bouncing ball using a Calculator Based Ranger (CBR). You will then analyze the collected data and attempt to model the variations in a bouncing ball's height as a function of time for one particular bounce.

YOU NEED:

1 CBR unit
1 Ball (racquetball or basketballs work well)



PRIOR TO THE ACTIVITY, MAKE THE FOLLOWING PREDICTIONS:

Make a prediction of the height from the floor as a function of time.

2. Make a prediction of the distance from the CBR as a function of time.

INSTRUCTIONS

Be sure the ball is bounced on a smooth surface. Do not allow anything to obstruct the path between the CBR and the ball while data is being collected.

Run RANGER on your calculator by selecting it the CBR/CBL app.

From the Applications menu of RANGER, Choose meters as the units.

Select 3:Ball Bounce.

Be sure to hold the CBR at least 1.5 meters from the Ball when collecting data.

ACTIVITY DATA

The resulting plot of distance versus time should appear to be a series of parabolic sections with decreasing maximum heights.

If you are dissatisfied with your results, press ENTER and choose Repeat Sample to recollect the data.

Once you are satisfied with the results, make a sketch of the height versus time plot. Include the units and values along each axis.

SELECTING THE DATA:

We will analyze the data for one parabola. From the LIST menu, arrow right to OPS and choose 8:Select(. The select feature allows you to choose a portion of the graph and place those data points into another set of lists. You type the lists where your want to place the new data separated by commas. For this activity, place the data in L_3 and L_4. Your home screen should show **Select(L_3,L_4).** Press ENTER and you will be taken to the graph. Move the cursor to the beginning of the parabola that you want to choose. Avoid the sharp point unless you are sure it is part of the parabola that you want. Press ENTER to mark this point. Move the cursor to the end of the parabola and press ENTER. The plot will now show the selected data which is located in L_3 and L_4.

Sketch the selected data in the space to the right.

Questions:

1. In this activity, the ball bounced straight up and down beneath the CBR, yet the data plot seems to depict a ball that is bouncing sideways. Explain why this is so.

2. Press [TRACE]. Move along the height versus time plot and estimate the x- and y-coordinates of the vertex of the parabola (in this case, the maximum point on the curve). Record these values in the table below.

vertex	
x-coordinate	**y-coordinate**

3. What do the x and y coordinates represent physically?

4. The theoretical model for the height vs time data is quadratic. We will attempt to fit our data with a quadratic function of the form:

$$y = A(x-B)^2 + C$$

where *b* is the x-coordinate of the vertex, *c* is the y-coordinate of the vertex, and *a* determines the parabola's *dilation* (stretch or spread). This model is sometimes called *vertex form*.

5. You will use an application, Transformation Graphing, to help you fit the equation to this data. Press the [APPS] key and select Transfrm.

   Press [Y=] and move the cursor to Y1. Enter the equation Y = A(X - B)^2 + C and then press [GRAPH]. Move the cursor the B on the screen and enter the value for the x-coordinate of the vertex. Move the cursor to the C and enter the value of the y-coordinate of the vertex.

6. To obtain a good fit, you will need to adjust the value of A. Use the method described above to store different numbers to the variable A. Record the A-value that works best in the space below:

   A = ______________

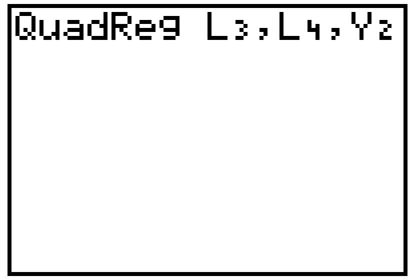
7. It is also possible to express any quadratic function in the *general form*, as described earlier:

$$y = ax^2 + bx + c$$

where the coefficient *a* is identical to that found in question (6) above, but *b* and *c* are different. To determine these coefficients, substitute the proper values for A, B, and C found above into the quadratic expression from question (2), expand it, and collect like terms. Record the corresponding values of a, b, and c in the table below:

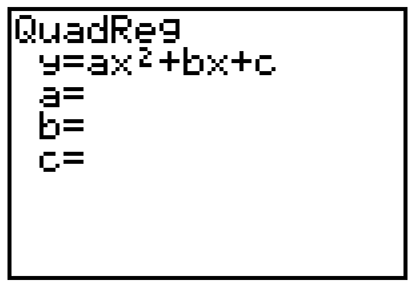
a	
b	
c	

8. The calculator has a built-in feature that allows it to compute the best-fitting quadratic equation through a set of data. To perform a quadratic regression on the data that you selected, press [STAT] arrow right to CALC. Select **QuadReg** to place the quadratic regression command on the home screen. Then press [2nd] [L3] [,] [2nd] [L4] [,] [VARS] arrow right to select YVARS and select Function and then Y2 [ENTER].



9. Copy the values that appear on your calculator screen into the matching table to the right.

   Are the values of a, b, and c in the quadratic regression equation above consistent with your table values from question (7)?



10. Press [Y=], move the cursor onto the equal sign for Y2 and press [ENTER] to turn on this equation. How well does it fit with your data?

11. In your own words, describe how the constant *a* affects the graph of $y = A(x-B)^2 + C$ Specifically, how does the sign of A change the graph?

12. Suppose you had chosen the parabolic section for the bounce just to the right of the one you actually used in this activity. Describe how each of the constants A, B and C would have to change, if at all, in order to fit this parabolic section with the equation $y = A(x-B)^2 + C$.

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13. From your physics lessons, you learned that for constant acceleration, the equation for motion is: $d = d_o + v_o t + \frac{1}{2}at^2$. How does this equation relate to the data that you collected? Relate each part of the equation and each variable.

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14. From your data, what is the value of the acceleration due to gravity?_________________

15. How close is this to the theoretical value? Find the percentage of error. Show your calculation.

16. Do the values of *b* and *c* from your regression equation represent the initial velocity and initial position of the ball? Explain your reasoning.

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