Cool It!

When you have a drink which is very hot, you have probably noticed that it quickly cools off to a temperature that you consider tolerable. Your drink then remains in a drinkable temperature range for quite a while until it eventually cools off too much as it approaches room temperature. Newton's law of cooling provides us with a theoretical model for this situation. It states that the temperature difference between a hot object and its surroundings decreases exponentially with time.

YOU NEED:
1 CBL2 Unit
1 TI-84 Calculator with Unit-to-Unit Link Cable
1 TI Temperature Probe
1 Cup of Hot Water

In this activity we will use the CBL and a temperature probe to collect data which will allow us to simulate the temperature variations that occur as a liquid is cooling. To do this we will place the probe in a cup of hot water and then remove it and allow it to cool in the air for one minute. We will then use the TI-84 calculator to fit a mathematical model to the data. In the box to the right, make a sketch of what you predict the graph of the data will look like.

PROCEDURE

1. Connect a Temperature Probe to CH 1 on the CBL 2 data collection device. Use the unit-to-unit cable to connect the data collection device to the TI Graphing Calculator. Firmly press in the cable ends.

2. Turn on the calculator and start the DATAMATE software. Press CLEAR to reset the software.

3. Set up DATAMATE for a temperature probe.

    a. Select SETUP from the main screen.
    b. Press ENTER to select CH1.
    c. Choose TEMPERATURE from the SELECT SENSOR list.
    d. Choose the type of temperature probe (with units of F) you are using from the list.
    e. Select OK to return to the main screen.

4. Set up the calculator and the data collection device.

    a. Select SETUP from the main screen.
    b. Press ▲ to select MODE and press ENTER.
    c. Select TIME GRAPH from the SELECT MODE menu to collect data as a function of time.

d. Select CHANGE TIME SETTINGS from the TIME GRAPH SETTINGS menu to set the data collection rate.
e. Enter 1 as the time between samples in seconds (remember to finish this and all numeric entries with (ENTER)).
f. Enter 60 as the number of samples.
g. Select OK twice to return to the main screen.

INSTRUCTIONS:
In this activity, we will observe temperature variations as the heated temperature probe is allowed to cool in the air. This will effectively simulate the action of a cooling cup of liquid while allowing us to collect the data in a relatively short period of time.

1. Get a cup of hot water to be used in this experiment. The water should be very hot, although it does not need to be boiling.
2. Place the probe in the cup of hot water and wait until the temperature readings on the screen of your calculator stabilize.
3. Remove the probe from the cup and press START.

ACTIVITY DATA:
Your data should show temperature values that decrease rapidly at first, then level off as the time values increase. If your data is not satisfactory, press (ENTER) to return to the main screen of DATAMATE and perform another trial. Begin again with step one from the INSTRUCTIONS section above. If your data is satisfactory, sketch a plot of your temperature vs. time data on the axes below.

To return to a graph of the data outside of DATAMATE

a. press (ENTER) to return to the main screen.

b. Select QUIT to leave DATAMATE, and then follow any instructions on the screen to return to the calculator's home screen.

c. Press (ZOOM)
d. Press (▼) until ZoomStat is highlighted
e. press (ENTER) to display a graph with *x* and *y* ranges set to fill the screen with data.

QUESTIONS:

1. The theoretical model for the cooling curve is exponential. We will attempt to fit our data with a curve of the form:
$$y = A\,B^x + C$$
where x is time and y is temperature. In this model, C represents room temperature, the value the modeling function is approaching as the curve flattens out. Press GRAPH, then press [2nd] [DRAW] and select Horizontal to put a horizontal marker on the screen. Use the arrow keys to move this marker up and down on the screen until it is resting at a point where you would visualize the temperature leveling out. Be sure all the recorded data points lie above this point. Record the y-value of your line as the room temperature C.

$$C = \underline{\hspace{3cm}}$$

2. The variable x represents time in our equation $y = A\,B^x + C$. Substituting $x = 0$, we arrive at $y = A + C$. Therefore, A equals the y-intercept or initial temperature minus C, the room temperature. Press [TRACE] to identify this initial value and record it to the nearest hundredth in the space below. Solve the equation for A, and record this value in the space provided:

   y-intercept = $\underline{\hspace{3cm}}$ $A = \underline{\hspace{4cm}}$

3. Press [Y=] and move the cursor to the first unused function register. Enter the expression $A*B{\wedge}X + C$ and then press [2nd] [QUIT] to return to the home screen. Enter the value of A found above then press [STO▸] [ALPHA] [A] [ENTER] to store this value to variable A on your calculator. Repeat this procedure to store variables C and B, starting with $B = 1$. Then press GRAPH to display the data and curve on the same screen. To obtain a good fit, you will need to adjust the value of B. Use the method described above to store different numbers for the variable B. View the graph for each new value of B that you test. Experiment until you find a value that provides a good fit for the data. Record the B value that works best in the space below:

   $B = \underline{\hspace{4cm}}$

4. Record your final equation on the space provided. $\underline{\hspace{6cm}}$

5. We can also fit this curve using an *exponential regression* on the TI-84 with a few modifications. The TI-84 uses the exponential model $y = a\,b{\wedge}x$ to fit the data. This model assumes that the curve approaches the line $y = 0$ as time values get larger and larger. In other words, the calculator assumes that $C = 0$. We can still use this model simply by applying a vertical shift of C units to our data. To achieve this, press [STAT] [ENTER] to go to the list editor. Use the arrow keys to move to the very

top of list L3 so that L3 is highlighted. Press ⟨2nd⟩ [L2] [-] [alpha] [C] ⟨ENTER⟩

What do the numbers in L3 represent? -

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To perform an exponential regression on the data in L1 and L3 and paste it into Y1, press ⟨STAT⟩ and arrow right to CALC. Select ExpReg to place the exponential regression command on the home screen. Press ⟨2nd⟩ [L1] [,]⟨2nd⟩ [L3] [,]⟨VARS⟩ and arrow right to YVARS. Select Function and Y2 to paste the regression command on the home screen and store the equation to Y2. Press [⟨ENTER⟩to the perform the regression.

What were your results. _____

Are the values of a and b in the exponential regression equation consistent with your results from the previous equations? _____
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6. Press ⟨Y=⟩ and use the arrow keys move to the end of the regression equation in Y2. Press [+] [alpha] [C] to adjust for the vertical shift you made earlier. Press ⟨GRAPH⟩to see the data and the 2 modeling curves. How well does each equation fit the data?

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7. *Describe how the value of B affects the temperature vs. time graph,* y = A B^x + C.

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8. Why must the value of B be lass than one? What shape would the graph have if B was greater than one?

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