Module 2 - Computer Algebra

Introduction

An important feature that sets the TI-89 apart from many other graphing calculators is its computer algebra system. A computer algebra system includes not only the ability to calculate numerical expressions and produce graphs, but it can also manipulate symbols and perform exact calculations, which can facilitate discovery learning. With this type of learning, you can experience the thrill that mathematicians enjoy when discovering a theorem.

Lesson Index:

After completing this module, you should be able to do the following:

- Perform exact calculations on the TI-89
- Understand the difference between AUTO, EXACT and APPROXIMATE modes
- Use the TI-89 catalog
- Define and delete variables
- Create activities in which your students use computer algebra to discover theorems

Lesson 2.1: Numerical Calculations

In this lesson you will use the TI-89 computer algebra system to perform exact and approximate calculations.

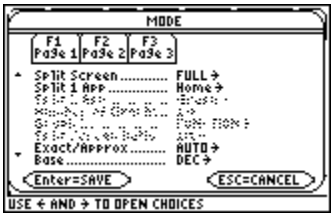
- Turn your calculator on and go to the Home screen by pressing HOME
- Clear the History Area and Edit Line by pressing F1 8 CLEAR

Mode Settings

Modes and their settings control how numbers, commands, and graphs are displayed and interpreted. The Mode dialog box is the place where you can change the mode settings.

- Display the Mode dialog box by pressing MODE

You can go to different pages within the dialog box by pressing F1, F2, and F3, and you can scroll up and down the pages with the cursor movement keys, ▲ and ▼.



The Exact/Approx Mode Setting

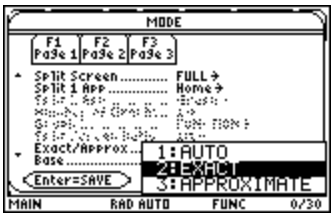
The Exact/Approx setting determines the way calculations are carried out and how the results of those calculations are displayed. By default, the Exact/Approx setting is AUTO, but you can select two other settings: EXACT or APPROXIMATE. All three settings are discussed in this lesson.

The EXACT Mode

In EXACT mode, all values entered as part of an expression are considered to be exact, all computations are done using exact methods, and all results are displayed in exact form (using fractions, square roots, etc.).

To select the EXACT mode,

- Go to page 2 of the Mode dialog box by pressing MODE F2
- Highlight Exact/Approx by using the down cursor movement key, ▼
- Display the Exact/Approx menu by pressing ▶
- Highlight EXACT by pressing ▼

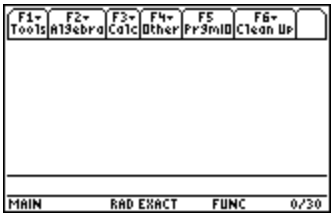


- Select EXACT and exit this menu by pressing ENTER
- Save the mode settings by pressing ENTER again

Tech Tip!

You must press ENTER to save the Mode settings and to exit the Mode dialog box. If you use some other key, such as ESC or HOME, the changes you made in the Mode settings will not be saved.

You should be on the Home screen. Notice one of the current Mode settings shown on the Status Line under the Edit Line is EXACT.



Now you are ready to perform exact calculations. If you make mistakes while entering the following calculations, you can erase with the ← key or clear the entire Edit Line and start over with CLEAR.

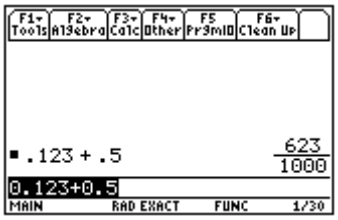
Exact Calculations of Fractions

- Find the exact value of $\frac{1}{2}+\frac{3}{4}$ by pressing 1 ÷ 2 + 3 ÷ 4 ENTER

After you press ENTER, the expression being evaluated appears on the left side of the History Area and the exact result appears on the right side. Any fractions in the result are written in reduced form.

Treatment of Decimal Numbers in Exact Mode

- Find the value of 0.123 + 0.5 in Exact Mode.

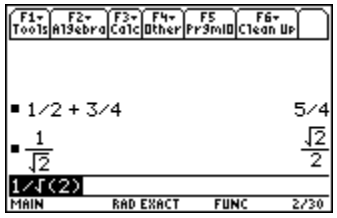


Notice that the TI-89 treats the decimal fractions 0.123 and 0.5 as exact.

Exact Calculations Involving Square Roots

The next calculation involves the square root function, which is a second function above the multiplication key in the right column of keys.

- Find the exact value of $\frac{1}{\sqrt{2}}$ by pressing [1] [÷] [2nd] [√] [2] [)] [ENTER]



The TI-89 returns $\frac{\sqrt{2}}{2}$ by rationalizing the denominator. The calculator will also reduce a fraction that is the result of rationalizing the denominator.

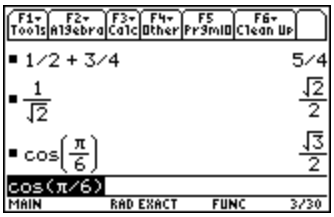
2.1.1 Find the exact value of $\sqrt{300}$ and $\sqrt{11}$.

Radian Mode

In the next calculation, you are going to find the value of a trig function. Your calculator should say "RAD" in the Status Line, indicating that you are in radian mode. Radian mode is one of the factory default settings selected when you reset to the factory setting in Lesson 1.1. If your calculator is not in radian mode, you can change the Angle setting to Radian on page 1 of the Mode dialog box.

Exact Calculations with Trig Functions

- Find the exact value of $\cos\left(\frac{\pi}{6}\right)$ by pressing [2nd] [COS] [2nd] [π] [÷] [6] [)] [ENTER].



The exact value of $\cos\left(\frac{\pi}{6}\right)$ is $\frac{\sqrt{3}}{2}$.

2.1.2 Compute $\cos\left(\frac{\pi}{9}\right)$ in Exact mode on the TI-89.

The APPROXIMATE Mode

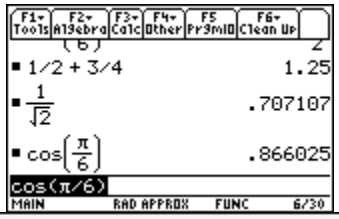
To see the difference between EXACT and APPROXIMATE modes, set your calculator to APPROXIMATE mode and then repeat the above calculations.

To change to APPROXIMATE mode,

- Return to page 2 of the Mode menu by pressing MODE F2
- Display the Exact/Approx menu by pressing ▼▼ then ▶
- Select APPROXIMATE by pressing 3
- Save the new settings by pressing ENTER

  Your calculator should say APPROX in the Status Line.

Repeat the three calculations you found in Exact mode: $\frac{1}{2}+\frac{3}{4}$, $\frac{1}{\sqrt{2}}$, and $\cos\left(\frac{\pi}{6}\right)$. The results are now decimal approximations.



Tech Tip!

Display Digits Factory Setting

The default setting for "Display Digits" is FLOAT 6, not FLOAT. If you reset your calculator to its default settings in Lesson 1.1, your calculator will display a maximum of six significant digits for any number in the result.

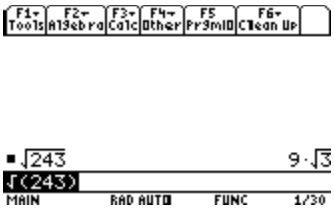
The AUTO Mode

AUTO is the other setting available in the Exact/Approx menu of the Mode dialog box. AUTO uses APPROXIMATE mode whenever the expression to be evaluated contains a decimal point. Otherwise, AUTO uses EXACT mode whenever possible. Certain higher order functions (e.g. "integrate" and "solve") may return APPROXIMATE results even if your entry does not contain a decimal point, but only when an EXACT answer cannot be found.

Calculating an Exact Value in AUTO Mode

To find $\sqrt{243}$ in AUTO mode,

- Clear the Home screen and Edit Line by pressing [F1] [8] [CLEAR]
- Change to AUTO mode by pressing [MODE] [F2] [▼] [▼] [▶] [1] [ENTER]
- Calculate $\sqrt{243}$ by pressing [2nd] [√] [2] [4] [3] [)] [ENTER]

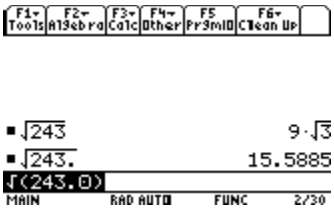


The TI-89 returns $9\sqrt{3}$, an exact result with a simplified radical, and AUTO is shown in the Status Line.

Calculating an Approximate Value in AUTO Mode

To calculate an approximate value of $\sqrt{243}$,

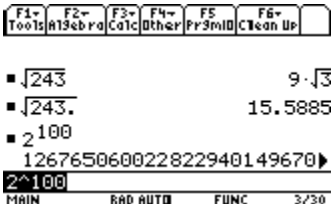
- Enter $\sqrt{243.0}$ by pressing [2nd] [√] [2] [4] [3] [.] [0] [)] [ENTER]



The result is approximate because the entered expression contained a decimal point.

Viewing Large Results

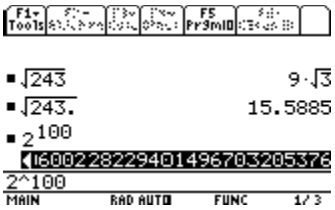
Compute 2^{100} by pressing [2][^][1][0][0][ENTER].



The TI-89 computer algebra system computes the exact answer, but the result is too big to fit on the screen, as indicated by the small arrow at the right of the result.

To see the part of the result that is hidden,

- Move up into the history area by pressing
- Scroll to the right by pressing



Now the first part of result is hidden, as indicated by the small arrow shown at the left of the result.

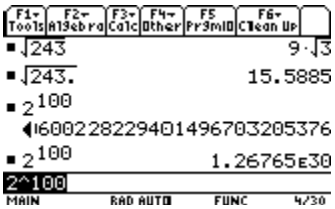
- Move back down to the Edit Line by pressing

Approximate Results Using [◆][ENTER]

Another way to obtain an approximate result is to press [◆][ENTER] at the end of a calculation instead of [ENTER]. Notice the approximate symbol (≈) in green above the [ENTER] key.

You should still have the expression 2^100 in the Edit Line.

- Find an approximate value for 2^{100} by pressing [◆][ENTER].



The approximate result is given in scientific notation.

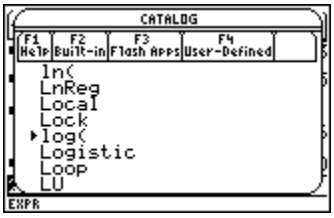
[◆][ENTER] also works in both AUTO and EXACT mode.

Calculating log *x*

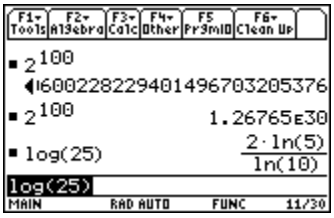
The last calculation in this lesson involves logarithms. The natural logarithm (ln) is a second function above the *x*-variable key, which is in the left column of keys. There is no dedicated key for the common logarithm (log), but it can be found in the catalog, which is an alphabetical listing of all the commands and functions available. Pressing CATALOG accesses the list. You could also type in log using the keyboard.

To find the value of log(25),

- Go to the catalog entries that begin with the letter "L" by pressing CATALOG 4. Notice that there is an "L" above the 4 key but that you don't need to press alpha before pressing 4.
- Move the cursor until it points to log(by repeatedly pressing ▼

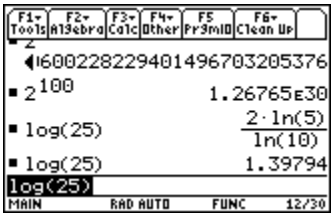


- Paste the log(function to the Edit Line by pressing ENTER
- Finish entering the expression by pressing 2 5) ENTER



The result, which is exact, is expressed in terms of natural logarithms.

- Press ◆ ENTER to see a decimal approximation



2.1.3 Find the exact and an approximate value of log(32).

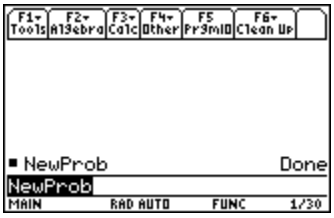
Lesson 2.2: Variables

In this lesson, you will learn to define and delete variables. It is important to know the difference between a defined and an undefined variable when working with algebraic expressions.

The "New Problem" Command

The Clean Up toolbar menu on the Home screen contains the "New Problem" command, which lets you start a new calculation from a cleared state without resetting the calculator's memory.

- Go to the Home screen and display the Clean Up menu by pressing [2nd][F6]
- Paste the New Prob command into the Edit Line by pressing [2] (or ▼ and [ENTER])
- Execute the command by pressing [ENTER]



The Home screen is cleared and all expressions and variables with one-letter names are deleted from memory, that is, "New Problem" *undefines* all one-letter variables.

Tech Tip!

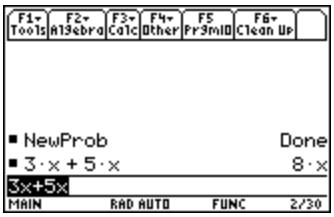
Defining Variables

When defining a variable that you want to retain beyond the current problem, use more than one character in the name. This prevents it from being deleted inadvertently when you use the "New Problem" command.

Simplifying Expressions

The computer algebra system simplifies the result of an expression that contains undefined variables.

- Enter the expression 3*x* + 5*x* by pressing [3][X][+][5][X][ENTER]



The variable *x* is undefined, so the calculator combines like terms and displays the result of 8*x*.

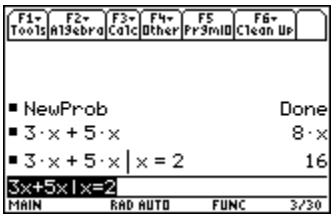
The With Command

The "with" command, which is accessed by pressing [|] in the left column of keys, can be used to evaluate an expression for a specific value of a variable without storing the value into the variable permanently.

Evaluate 3*x* + 5*x* with *x* = 2 by using the "with" command.

With the expression 3*x* + 5*x* in the Edit line,

- Modify the entry to evaluate the expression with *x* = 2 by pressing [▶] [|] [X] [=] [2] [ENTER]



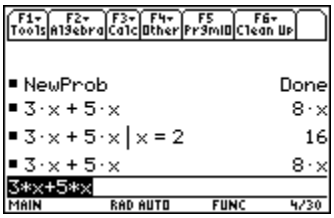
The variable *x* is temporarily defined to be 2 in the expression 3*x* + 5*x* but will be undefined in future entries.

Copying from the History Area

When working with complicated expressions and equations, you need only enter the expression once. You can then copy it from the History Area instead of retyping the entry.

To copy the expression 3*x* + 5*x* from the History Area and paste it to the Edit Line,

- Move the cursor up in the History Window and highlight 3*x* + 5*x* by pressing [▲] repeatedly
- Paste the expression to the Edit Line by pressing [ENTER]
- Evaluate the entry shown in the Edit Line by pressing [ENTER]



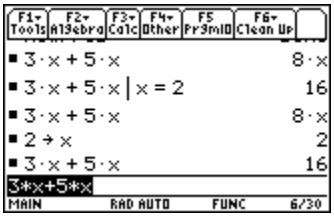
Notice that the value of the variable *x* is no longer 2.

Storing a Value into a Variable

Sometimes it will be convenient to define a variable to be a specific value. Define *x* to be 2 for future calculations by using the STO▶ key.

- Store 2 into *x* by pressing 2 STO▶ X ENTER
- Copy the expression 3*x* + 5*x* to the Edit Line by using ▲ to highlight it in the History Area again and press ENTER

Alternatively, you can use F4 [1:Define] X = 2 ENTER to define *x* to be 2.



The result was calculated as 3(2) + 5(2) because *x* is defined to be 2. The value of *x* will be 2 in all future calculations until a new value is stored in *x* or until the variable *x* is undefined.

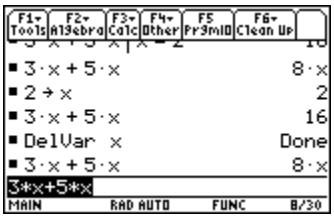
Undefining a Variable

You can delete the value stored in a variable by using the DelVar command, which is found in the F4 menu of the Home screen. This command undefines a variable.

- Paste DelVar into the Edit Line by pressing F4 4
- Undefine the variable *x* by pressing X ENTER

Now *x* is undefined and contains no value.

Evaluate 3*x* + 5*x* again to see that the result is 8*x*.



The variable *x* would also become undefined if the NewProb command was used.

2.2.1 Evaluate the expression $2x^2 - 3x$ with $x = 5b$.

Lesson 2.3: Pattern Recognition

In this lesson you will use a computer algebra system to create a mathematical laboratory where experiments are conducted, patterns are discovered, and conjectures are made. This type of inductive learning fosters a sense of ownership and interest in the concept discovered. Discovering patterns and making conjectures can motivate learning about proof.

Factoring the Difference of Two Squares

This example illustrates inductive, or discovery, learning. The concept to be developed is factoring the difference of two squares. In later modules, lessons that discover calculus concepts are developed.

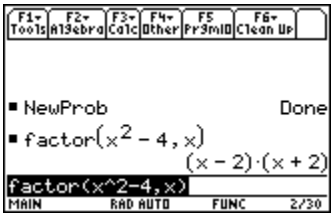
The procedure we will use to discover patterns is

- Explore several related examples
- Describe verbally the pattern of the result
- Predict more results
- Test the prediction
- Extend the type of examples explored
- Generalize

Explore Factoring $x^2 - 4$

To factor $x^2 - 4$,

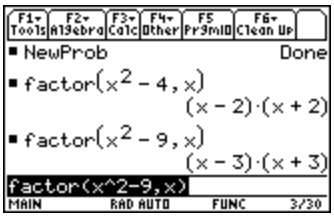
- Clear the Home Screen and all one-letter variables with the New Problem command by pressing [2nd] [F6] [2] [ENTER]
- Paste the factor(command to the Edit Line by pressing [F2] [2]
- Complete the command factor(x^2–4,x) by pressing [X] [^] [2] [–] [4] [,] [X] [)] [ENTER]



The factors of $x^2 - 4$ are $x - 2$ and $x + 2$.

Explore Factoring $x^2 - 9$

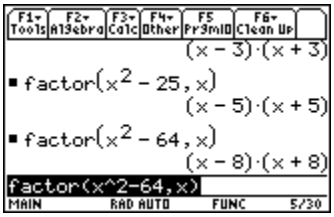
- Edit the expression in the Edit Line by pressing ▷ to move the cursor to the end of the expression and then by pressing ◁◁◁ [←] [9] to erase the 4 and replace it with 9
- Press [ENTER] to see the result



You may want to factor other examples of the difference of two squares to determine a pattern relating the original expression and its factors. After a few examples, you will probably notice a pattern. Can you predict the result of the next two commands before entering them? Write your predictions down before entering the commands into the calculator.

Explore by Factoring Other Examples

- factor(x^2 – 25, *x*)
- factor(x^2 – 64, *x*)



Describe the Pattern Verbally

2.3.1 When you see the pattern, describe it verbally and express it with an algebraic expression.

Predict the Pattern of the Factors of The Difference of Two Squares

2.3.2 Predict the factors of $x^2 - a^2$.

Test the Prediction

2.3.3 Check your prediction.

- Factor $x^2 - a^2$ by using the factor(command.

  Remember "a" is an ALPHA feature above the = key.

Extend and Check

2.3.4 Predict the factors of $x^2 - 5$.

2.3.5 Factor $x^2 - 5$ by using the factor command.

Generalize

2.3.6 Predict the factors of $x^2 - a$.

Tech Tip!

Factoring Over Different Number Sets

The command factor(x^2–5,x) gives a different result than factor(x^2–5). The first command factors over the real numbers. The second command factors over the rational numbers. The command cfactor(factors over the complex numbers. This command is found in the F2:Algebra, A:Complex submenu. You could also type the entire command or add a "c" in front of the factor(command.

2.3.7 Try cfactor(x^2+4,x).

Binomial Expansion

You will now explore expanding expressions like $(x + 1)^n$, which are called binomial expansions. The pattern of the powers of the variable in the expansion is

$$a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$$

We want to find the values of the coefficients. In the following exploration you should look for a pattern in these coefficients and how they relate to the power of the expansion, *n*.

- Start fresh by selecting NewProb from the Clean Up menu on the Home screen by pressing [2nd] [F6] [2] [ENTER]

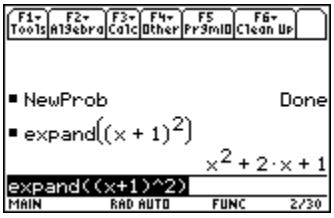
The Expand(Command

To explore binomial expansions, you will use the expand(command, which is in the [F2] Algebra menu of the Home screen.

Explore the Expansion of $(x + 1)^2$

To expand the expression $(x + 1)^2$,

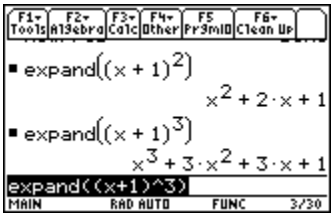
- Paste the expand(command to the Edit Line by pressing [F2] [3]
- Complete the command expand ($(x + 1)^2$) by pressing [(] [X] [+] [1] [)] [^] [2] [)] [ENTER]



When n = 2, the coefficients of the expansion are 1, 2, 1.

Explore the Expansion of $(x + 1)^3$

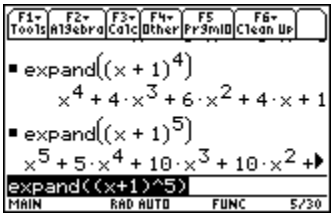
- Edit the expression in the Edit Line to cube the binomial by pressing ◀ ◀ ← 3 ENTER



When n = 3, the coefficients are 1, 3, 3, 1.

Explore the Expansion of $(x + 1)^4$ and the Expansion of $(x + 1)^5$

- Raise the binomial to the fourth power with ◀ ◀ ← 4 ENTER
- Edit the expression in the Edit Line to raise the binomial to the fifth power



When n = 4, the coefficients are 1, 4, 6, 4, 1.

When n = 5, the coefficients are 1, 5, 10, 10, 5, 1.

Describe the Pattern

The coefficients found by expanding $(x + 1)^n$, when n = 2,3,4,5 are shown in the respective rows of the triangle below.

1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

2.3.8 Describe the pattern shown in the triangle above.

You can expand the triangle, both below and above the existing rows, and use the new rows to determine the coefficients of other expansions of $(x + 1)^n$.

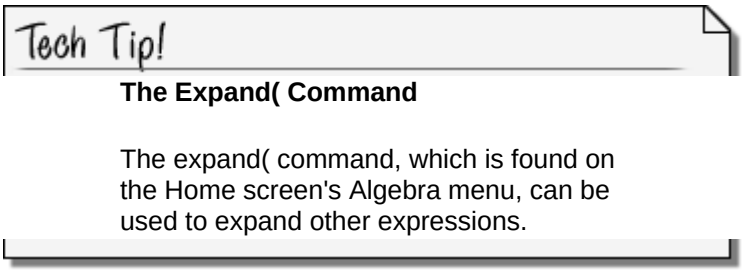
Predict $(x + 1)^6$ and Confirm

2.3.9 Use the next line below the triangle shown above to predict $(x + 1)^6$ and then check your answer with the expand(command.

We will not extend and generalize this topic at this time.

Discovery Learning

Using the TI-89, you are able to see the results of many binomial expansions more quickly than would be possible with only paper and pencil. This facilitates inductive learning. The Explore, Describe, Predict, Confirm, Extend, and Generalize procedure is a more active form of learning than reading or listening. Hopefully, when you use this procedure, you have more of a sense of discovering the theorem, rather than just being told the result.



2.3.10 Expand the product $(x + 1)(x^2 - x + 1)$ by using the expand(command.

Self Test

1. Use your calculator to find the exact value of $\sin\left(\frac{\pi}{12}\right)$.
2. Use your calculator to express $\frac{1}{\sqrt{735}}$ in simplest radical form.
3. Which of the following modes can produce exact results?
   A. Auto
   B. Exact
   C. Approximate
4. True or False: The following calculation will produce an exact result in Exact mode:

   2 ÷ 3 + 1 ♦ ENTER

5. If you couldn't remember which menu contains the DelVar(command, you could look in the catalog. What keystroke sequence would you use to jump to the commands that begin with "D" in the catalog?
6. What is the difference between using the [I] key and the [STO▶] key to assign a value to a variable ?
7. What are some advantages of inductive or discovery lessons?